# Section 9.3

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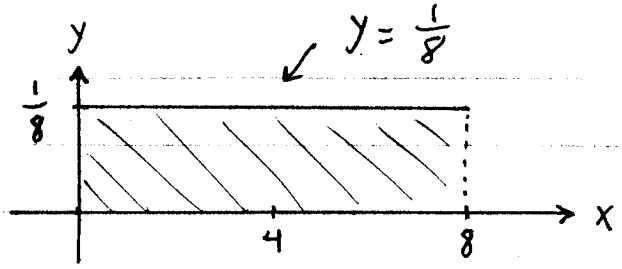
1.)




a.) $\mu = \int_0^8 x\left(\frac{1}{8}\right) dx = \frac{1}{16}x^2\Big|_0^8 = 4$

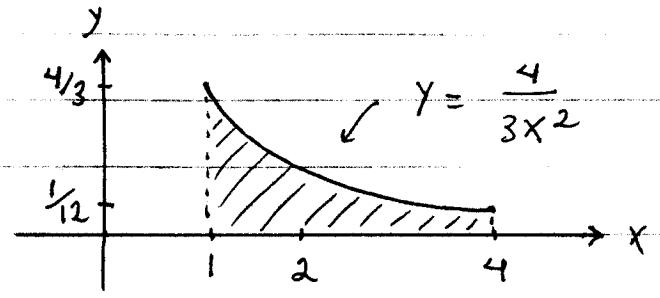
b.) $\int_0^m \frac{1}{8}\, dx = \frac{1}{8}x\Big|_0^m = \frac{m}{8} = \frac{1}{2} \rightarrow \boxed{m = 4}$

c.) $V(x) = \int_0^8 x^2\left(\frac{1}{8}\right) dx - \mu^2$

$= \frac{1}{24}x^3\Big|_0^8 - (4)^2 \approx \boxed{5.333}$

d.) $\sigma = \sqrt{V(x)} \approx \sqrt{5.333} \approx \boxed{2.31}$

4.)




a.) $\mu = \int_1^4 x\left(\frac{4}{3x^2}\right) dx = \int_1^4 \frac{4}{3}\cdot\frac{1}{x}\, dx = \frac{4}{3}\ln x\Big|_1^4$

$= \frac{4}{3}\ln 4 - \frac{4}{3}\ln 1 = \frac{4}{3}\ln 4 \approx \boxed{1.848}$

b.) $V(x) = \int_1^4 x^2\left(\frac{4}{3x^2}\right) dx - \mu^2 = \int_1^4 \frac{4}{3}\, dx - (1.848)^2$

$= \frac{4}{3}x\Big|_1^4 - (1.848)^2 \approx \boxed{0.585}$

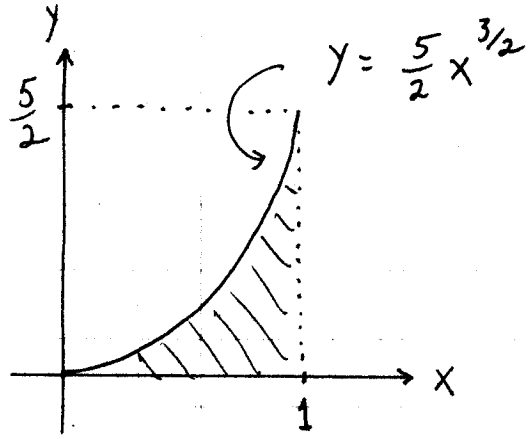
c.) $\int_1^m \frac{4}{3x^2}\,dx = \frac{4}{3}\cdot\frac{-1}{x}\Big|_1^m = \frac{-4}{3m} - \frac{-4}{3} = \frac{1}{2} \rightarrow$

$\frac{-4}{3m} = \frac{1}{2} - \frac{4}{3} \rightarrow \frac{-4}{3m} = \frac{-5}{6} \rightarrow -24 = -15m \rightarrow$

$m = \frac{24}{15} = \boxed{\frac{8}{5}}$

d.) $\sigma = \sqrt{V(x)} \approx \sqrt{0.585} \approx \boxed{0.765}$

5.)




a.) $\mu = \int_0^1 x\left(\frac{5}{2}x^{3/2}\right)dx = \int_0^1 \frac{5}{2}x^{5/2}\,dx$

$= \frac{5}{2}\cdot\frac{2}{7}x^{7/2}\Big|_0^1 = \boxed{\frac{5}{7}} \approx 0.714$

b.) $V(x) = \int_0^1 x^2\left(\frac{5}{2}x^{3/2}\right)dx - \mu^2$

$= \int_0^1 \frac{5}{2}x^{7/2}\,dx - \left(\frac{5}{7}\right)^2 = \frac{5}{2}\cdot\frac{2}{9}x^{9/2}\Big|_0^1 - \left(\frac{5}{7}\right)^2$

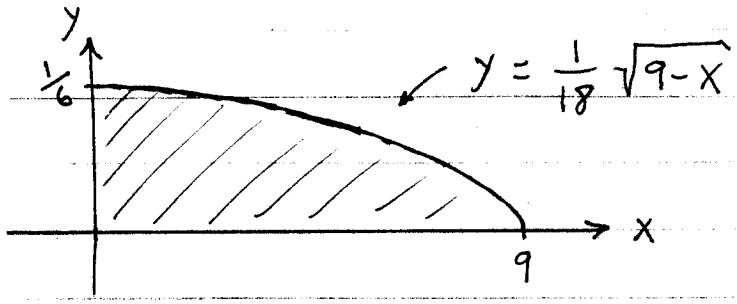
$= \frac{5}{9} - \left(\frac{5}{7}\right)^2 \approx \boxed{0.045}$

c.) $\int_0^m \frac{5}{2}x^{3/2}\,dx = \frac{5}{2}\cdot\frac{2}{5}x^{5/2}\Big|_0^m = m^{5/2} = \frac{1}{2} \rightarrow$

$m = \left(\frac{1}{2}\right)^{2/5} \approx \boxed{0.758}$

d.) $\sigma = \sqrt{V(x)} \approx \sqrt{0.045} \approx \boxed{0.212}$

10.)




a.) $\mu = \int_0^9 x \cdot \frac{1}{18}\sqrt{9-x}\,dx$ (Let $u = 9-x$, $du = -dx \rightarrow -du = dx$ and $x = 9-u$, $x: 0 \rightarrow 9$ so $u: 9 \rightarrow 0$)

$= -\int_9^0 (9-u)\cdot\frac{1}{18}u^{1/2}\,du = \frac{-1}{18}\int_9^0 (9u^{1/2} - u^{3/2})\,du$

$= \frac{-1}{18}\left(9\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_9^0 = 0 - \frac{-1}{18}\left(\frac{18}{3}\cdot 9^{3/2} - \frac{2}{5}9^{5/2}\right)$

$= \frac{1}{18}\left(\frac{18}{3}\cdot 27 - \frac{2}{5}\cdot 243\right) = 3.6$

b.) $V(x) = \int_0^9 x^2\cdot\frac{1}{18}\sqrt{9-x}\,dx - \mu^2$ (Let $u = 9-x \rightarrow du = -dx \rightarrow -du = dx$ and $x = 9-u$, $x: 0 \rightarrow 9$, $u: 9 \rightarrow 0$)

$= \frac{-1}{18}\int_9^0 (9-u)^2 u^{1/2}\,du - \mu^2$

$= \frac{-1}{18}\int_9^0 (81u^{1/2} - 18u^{3/2} + u^{5/2}) - \mu^2$

$= \frac{-1}{18}\left(81\cdot\frac{2}{3}u^{3/2} - 18\cdot\frac{2}{5}u^{5/2} + \frac{2}{7}u^{7/2}\right)\Big|_9^0 - \mu^2$

$= 0 - \frac{-1}{18}\left(54\cdot 9^{3/2} - \frac{36}{5}\cdot 9^{5/2} + \frac{2}{7}\cdot 9^{7/2}\right) - (3.6)^2 = 5.554$

c.) $\int_0^m \frac{1}{18}\sqrt{9-x}\,dx = \frac{-1}{18}\cdot\frac{2}{3}\cdot(9-x)^{3/2}\Big|_0^m$

$= \frac{-1}{27}(9-m)^{3/2} - \frac{-1}{27}(9)^{3/2} = 1 - \frac{1}{27}(9-m)^{3/2} = \frac{1}{2} \rightarrow$

$$\frac{1}{2} = \frac{1}{27}(9-m)^{3/2} \rightarrow \frac{27}{2} = (9-m)^{3/2} \rightarrow$$

$$\left(\frac{27}{2}\right)^{2/3} = 9-m \rightarrow m = 9 - \left(\frac{27}{2}\right)^{2/3} \approx \boxed{3.333}$$

d.) $\sigma = \sqrt{V(x)} \approx \sqrt{5.554} \approx \boxed{2.357}$

11.) $$\int_0^m \frac{1}{9} e^{-t/9}\, dt = -e^{-t/9}\Big|_0^m$$

$$= -e^{-m/9} - -e^{0} = 1 - e^{-m/9} = \frac{1}{2} \rightarrow$$

$$\frac{1}{2} = e^{-m/9} \rightarrow \ln\frac{1}{2} = \ln e^{-m/9} \rightarrow$$

$$\ln\frac{1}{2} = \frac{-m}{9} \rightarrow \boxed{m = -9\ln\frac{1}{2}}.$$

35.) a.) $\mu = E(x) = \int_0^6 x \cdot \frac{1}{36}x(6-x)\,dx = \int_0^6 \left(\frac{1}{6}x^2 - \frac{1}{36}x^3\right) dx$

$= \left(\frac{1}{18}x^3 - \frac{1}{144}x^4\right)\Big|_0^6 = \boxed{3}$ ;

$V(x) = \int_0^6 x^2 \cdot \frac{1}{36}x(6-x)\,dx - \mu^2 = \int_0^6 \left(\frac{1}{6}x^3 - \frac{1}{36}x^4\right) dx - \mu^2$

$= \left(\frac{1}{24}x^4 - \frac{1}{180}x^5\right)\Big|_0^6 - (3)^2 = \boxed{1.8}$ and $\boxed{\sigma \approx 1.342}$

b.) $\int_0^m \frac{1}{36}x(6-x)\,dx = \int_0^m \left(\frac{1}{6}x - \frac{1}{36}x^2\right) dx = \left(\frac{1}{12}x^2 - \frac{1}{108}x^3\right)\Big|_0^m$

$= \frac{m^2}{12} - \frac{m^3}{108} = \frac{1}{2} \rightarrow m^3 - 9m^2 + 54 = 0 \rightarrow$

$\boxed{m = 3}$ works.

c.) $P(\mu - \sigma < x < \mu + \sigma) = P(1.658 < x < 4.342)$

$= \int_{1.658}^{4.342} \frac{1}{36}x(6-x)\,dx = \left(\frac{1}{12}x^2 - \frac{1}{108}x^3\right)\Big|_{1.658}^{4.342} \approx \boxed{0.626}$

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42.) $f(x) = 0.04$ on $[0, 25]$ :

a.) $\mu = E(x) = \int_0^{25} x\,(0.04)\,dx = 0.02x^2\Big|_0^{25} = \boxed{\frac{25}{2}}$

b.) $\int_0^m (0.04)\,dx = \frac{1}{2} \rightarrow 0.04x\Big|_0^m = \frac{1}{2} \rightarrow 0.04m = \frac{1}{2}$

$\rightarrow m = \boxed{\frac{25}{2}}$.

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43.) $f(x) = 4 - 8x$ on $[0, \frac{1}{2}]$ :

a.) $\mu = E(x) = \int_0^{1/2} x(4-8x)\,dx = \int_0^{1/2} (4x - 8x^2)\,dx$

$= \left(2x^2 - \frac{8}{3}x^3\right)\Big|_0^{1/2} = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

b.) $\int_0^m (4-8x)\,dx = \frac{1}{2} \rightarrow (4x - 4x^2)\Big|_0^m = \frac{1}{2} \rightarrow$

$4m - 4m^2 = \frac{1}{2} \rightarrow 0 = 8m^2 - 8m + 1 \rightarrow$

$m = \frac{8 \pm \sqrt{64-32}}{16} = \frac{8 \pm \sqrt{32}}{16} = \frac{8 \pm 4\sqrt{2}}{16} \rightarrow \boxed{m = \frac{2+\sqrt{2}}{4}}$