Section 4.4

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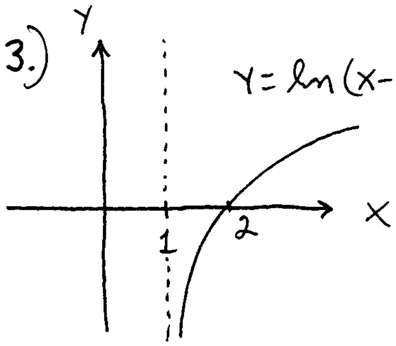
1.) $\ln 2 = 0.6931\ldots$ means $e^{0.6931\ldots} = 2$

7.) $e^{-3} = 0.0498\ldots$ means $\ln 0.0498\ldots = -3$

13.)



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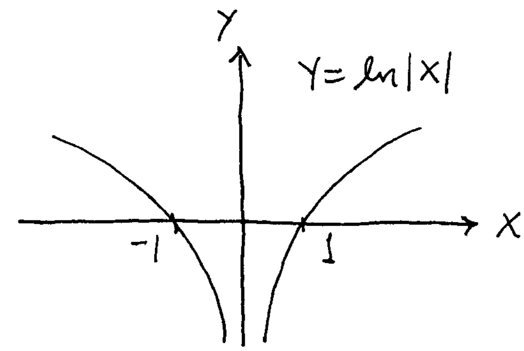

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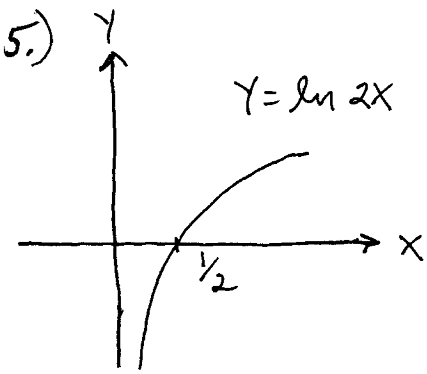

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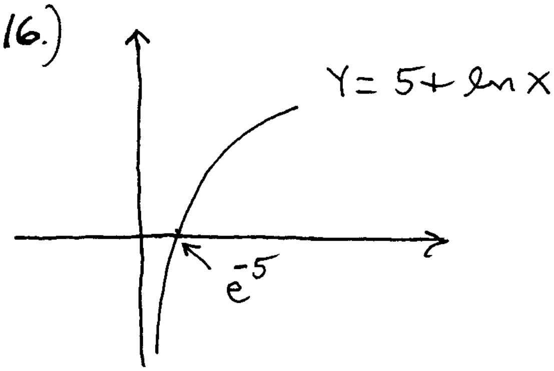

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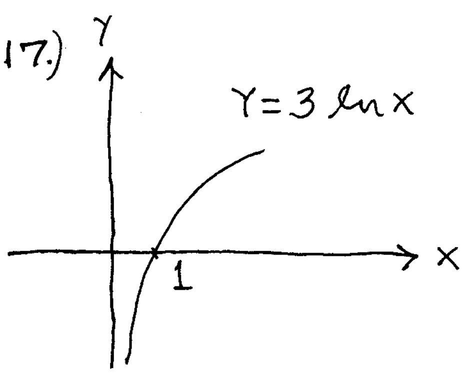

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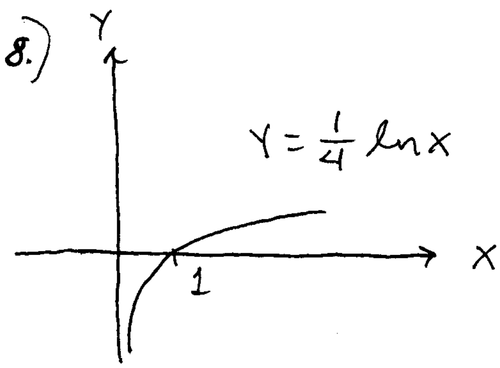

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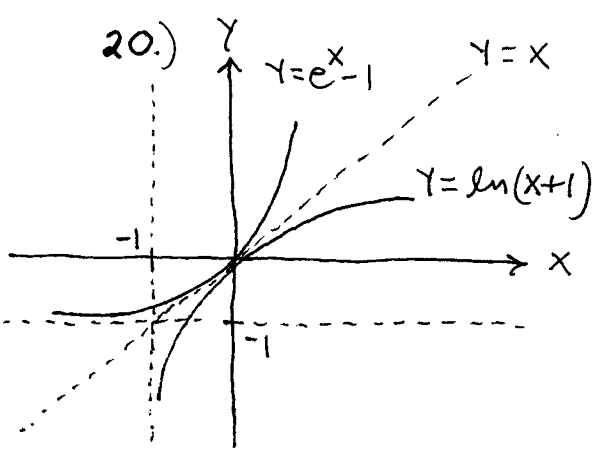


analytically: $Y = e^X - 1$
(switch) $X = e^Y - 1 \rightarrow$
$X + 1 = e^Y \rightarrow \ln(X+1) = \ln e^Y \rightarrow$
$\ln(X+1) = Y$, so
$Y = e^X - 1$ and $Y = \ln(X+1)$
are inverse functions

24.) $\ln e^{2x-1} = (2x-1)\ln e = (2x-1)(1) = 2x-1$

25.) $e^{\ln(5x+2)} = 5x+2$

30.) a.) $\ln 0.25 = \ln \frac{1}{4} = \ln 1 - \ln 4 = 0 - \ln 2^2$
$= -2\ln 2 \approx -2(0.6931) = -1.3862$

b.) $\ln 24 = \ln 3 \cdot 2^3 = \ln 3 + \ln 2^3 = \ln 3 + 3\ln 2$
$\approx 1.0986 + 3(0.6931) = 3.1779$

c.) $\ln 12^{1/3} = \frac{1}{3}\ln 3 \cdot 2^2 = \frac{1}{3}(\ln 3 + 2\ln 2)$
$\approx \frac{1}{3}(1.0986 + 2(0.6931)) \approx 0.8283$

d.) $\ln \frac{1}{72} = \ln 1 - \ln 2^3 \cdot 3^2 = 0 - (3\ln 2 + 2\ln 3)$
$= -(3(0.6931) + 2(1.0986)) = -4.2765$

39.) $\ln \frac{(3x)(x+1)}{(2x+1)^2} = \ln (3x)(x+1) - \ln (2x+1)^2$
$= \ln 3x + \ln(x+1) - 2\ln(2x+1) = \ln 3 + \ln x + \ln(x+1) - 2\ln(2x+1)$

43.) $3\ln x + 2\ln y - 4\ln z = \ln x^3 + \ln y^2 - \ln z^4$
$= \ln x^3y^2 - \ln z^4 = \ln \frac{x^3y^2}{z^4}$

46.) $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2-1)]$
$= \frac{2}{3}\ln(x+3) + \frac{1}{3}\ln x - \frac{1}{3}\ln(x^2-1)$
$= \ln(x+3)^{2/3} + \ln x^{1/3} - \ln(x^2-1)^{1/3}$
$= \ln (x+3)^{2/3}(x^{1/3}) - \ln(x^2-1)^{1/3}$
$= \ln \frac{(x+3)^{2/3} \cdot x^{1/3}}{(x^2-1)^{1/3}}$

54.) $2\ln x = 4 \rightarrow \ln x = 2 \rightarrow x = e^2$

57.) $300e^{-0.2t} = 700 \rightarrow e^{-0.2t} = \frac{7\cancel{00}}{3\cancel{00}} \rightarrow$

$\ln e^{-0.2t} = \ln(7/3) \rightarrow -0.2t = \ln(7/3) \rightarrow$

$t = \frac{\ln(7/3)}{-0.2} \approx -4.236$

60.) $2e^{-x+1} - 5 = 9 \rightarrow 2e^{-x+1} = 14 \rightarrow$

$e^{-x+1} = 7 \rightarrow \ln e^{-x+1} = \ln 7 \rightarrow$

$-x+1 = \ln 7 \rightarrow x = 1 - \ln 7 \approx -0.946$

62.) $\frac{50}{1+12e^{-0.02x}} = 10.5 \rightarrow$

$1+12e^{-0.02x} = \frac{50}{10.5} \rightarrow 12e^{-0.02x} = \frac{50}{10.5} - 1 \rightarrow$

$e^{-0.02x} = \frac{1}{12}\left(\frac{50}{10.5} - 1\right) \rightarrow \ln e^{-0.02x} = \ln \frac{1}{12}\left(\frac{50}{10.5} - 1\right)$

$\rightarrow -0.02x = \ln \frac{1}{12}\left(\frac{50}{10.5} - 1\right)$

$\rightarrow \quad x = \frac{1}{-0.02} \cdot \ln \frac{1}{12}\left(\frac{50}{10.5} - 1\right) \approx 60.00$

64.) $2^{1-x} = 6 \rightarrow \ln 2^{1-x} = \ln 6 \rightarrow$

$(1-x)\ln 2 = \ln 6 \rightarrow 1 - x = \frac{\ln 6}{\ln 2} \rightarrow$

$x = 1 - \frac{\ln 6}{\ln 2} \approx -1.585$

67.) $1000\left(1 + \frac{0.07}{12}\right)^{12t} = 3000 \rightarrow$

$$\left(1+\frac{0.07}{12}\right)^{12t} = \frac{3000}{1000} \longrightarrow$$

$$\ln\left(1+\frac{0.07}{12}\right)^{12t} = \ln 3 \longrightarrow$$

$$12t \cdot \ln\left(1+\frac{0.07}{12}\right) = \ln 3 \longrightarrow$$

$$t = \frac{\ln 3}{12 \ln\left(1+\frac{0.07}{12}\right)} \approx 15.74$$

73.) $P = 821.95\, e^{0.0358t}$

a.) $t = 20$ (2000) $\rightarrow P \approx 1681.9$ or

1,681,900 people

b.) 2,500,000 people $\rightarrow P = 2500 \rightarrow$

$$2500 = 821.95\, e^{0.0358t} \longrightarrow$$

$$e^{0.0358t} = \frac{2500}{821.95} \rightarrow \ln e^{0.0358t} = \ln\left(\frac{2500}{821.95}\right)$$

$$\rightarrow 0.0358t = \ln\left(\frac{2500}{821.95}\right)$$

$$\rightarrow t = \frac{1}{0.0358} \cdot \ln\left(\frac{2500}{821.95}\right) \approx 31 \text{ years}$$

so year is $1980 + 31 = 2011$

79.) $S = 80 - 14\ln(t+1)$

a.) $t = 0 \rightarrow S = 80 - 14\overset{0}{\cancel{\ln 1}} = 80$

b.) $t = 4 \rightarrow S = 80 - 14\ln 5 \approx 57.47$

c.) $S = 46 \rightarrow 46 = 80 - 14\ln(t+1) \rightarrow$
$14\ln(t+1) = 34 \rightarrow \ln(t+1) = \frac{34}{14} = \frac{17}{7} \rightarrow$
$t+1 = e^{17/7} \rightarrow t = e^{17/7} - 1 \approx 10.34$ mo.

$f(x) = \ln x$

85.) $f(0) = 0 \rightarrow \ln 0 = 0$ FALSE

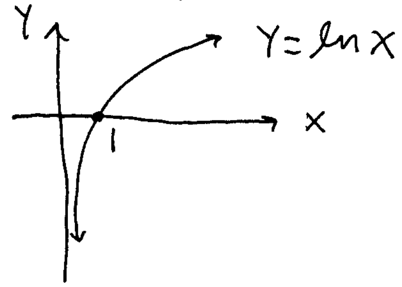
86.) $a > 0, x > 0,\ f(ax) = f(a) + f(x) \rightarrow$
$\ln(ax) = \ln a + \ln x$ TRUE

87.) $x > 2,\ f(x-2) = f(x) - f(2) \rightarrow$
$\ln(x-2) = \ln x - \ln 2$ FALSE

88.) $\sqrt{f(x)} = \frac{1}{2}f(x) \rightarrow \sqrt{\ln x} = \frac{1}{2}\ln x \rightarrow$
$(\ln x)^{1/2} = \frac{1}{2}\ln x$ FALSE

89.) If $f(u) = 2f(v) \rightarrow \ln u = 2\ln v \rightarrow$
$\ln u = \ln v^2 \rightarrow e^{\ln u} = e^{\ln v^2} \rightarrow u = v^2$
FALSE

90.) If $f(x) < 0 \rightarrow \ln x < 0 \rightarrow$



$0 < x < 1$ TRUE

Math 16B
Kouba
Solving Algebraic Logarithm and Exponential equations

1.) Solve for t .

a.) $e^t = 3$

b.) $e^{4-(1/3)t} = 2$

c.) $e^{t^2-3t-4} = 1$

d.) $e^{t+3} - e^t = 1/2$

e.) $e^{2t} - e^t = 0$

f.) $e^{2t} - 2e^t - 3 = 0$

g.) $2e^{2t} + e^t = 6$

h.) $\dfrac{e^t}{e^t - 2} = \dfrac{e^t - 1}{e^t + 3}$

2.) Solve for t .

a.) $\ln t = -1/2$

b.) $\ln(4 - t) = 7$

c.) $\ln(t^2 + t) = \ln 2$

d.) $\ln t + \ln(t + 2) = 0$

e.) $\ln(t + 1) - \ln(t - 1) = 1$

f.) $(\ln t)^2 - \ln t - 2 = 0$

g.) $(\ln t)^3 - \ln t = 0$

h.) $\dfrac{\ln t}{3 + \ln t} = \dfrac{\ln t + 1}{2 \ln t + 1}$

Algebraic Equations

1.) a.) $e^t = 3 \rightarrow \ln e^t = \ln 3 \rightarrow t = \ln 3$

b.) $e^{4-\frac{1}{3}t} = 2 \rightarrow \ln e^{4-\frac{1}{3}t} = \ln 2$

$\rightarrow 4 - \frac{1}{3}t = \ln 2 \rightarrow \frac{1}{3}t = 4 - \ln 2$

$\rightarrow t = 12 - 3\ln 2$

c.) $e^{t^2-3t-4} = 1 \rightarrow \ln e^{t^2-3t-4} = \ln 1$

$\rightarrow t^2 - 3t - 4 = 0 \rightarrow (t-4)(t+1) = 0$

$\rightarrow t = 4$ or $t = -1$

d.) $e^{t+3} - e^t = \frac{1}{2} \rightarrow e^3 e^t - e^t = \frac{1}{2}$

$\rightarrow e^t(e^3 - 1) = \frac{1}{2} \rightarrow e^t = \dfrac{\frac{1}{2}}{e^3 - 1}$

$\rightarrow e^t = \dfrac{1}{2(e^3-1)} \rightarrow \ln e^t = \ln\left(\dfrac{1}{2(e^3-1)}\right)$

$\rightarrow t = \ln\left(\dfrac{1}{2(e^3-1)}\right)$

e.) $e^{2t} - e^t = 0 \rightarrow (e^t)^2 - e^t = 0$

$\rightarrow e^t(e^t - 1) = 0 \rightarrow e^t = 0$ (NO!) or

$e^t - 1 = 0 \rightarrow e^t = 1 \rightarrow t = 0$

f.) $e^{2t} - 2e^t - 3 = 0 \rightarrow (e^t)^2 - 2(e^t) - 3 = 0$

$\rightarrow (e^t-3)(e^t+1)=0$

$\downarrow \quad \hookrightarrow e^t=-1$ (NO!)

$e^t=3 \rightarrow t=\ln 3$

g.) $2e^{2t}+e^t=6 \rightarrow 2(e^t)^2+(e^t)-6=0$

$\rightarrow (2e^t-3)(e^t+2)=0$

$\hookrightarrow e^t=-2$ (NO!)

$\downarrow$

$e^t=\frac{3}{2} \rightarrow t=\ln \frac{3}{2}$

h.) $\dfrac{e^t}{e^t-2} = \dfrac{e^t-1}{e^t+3} \rightarrow$

$e^t(e^t+3) = (e^t-2)(e^t-1) \rightarrow$

$\cancel{e^{2t}}+3e^t = \cancel{e^{2t}}-e^t-2e^t+2 \rightarrow$

$6e^t=2$

2.) a.) $\ln t = \frac{-1}{2} \rightarrow e^{\ln t} = e^{-1/2} \rightarrow t=e^{-1/2}$

b.) $\ln(4-t)=7 \rightarrow e^{\ln(4-t)}=e^7$

$\rightarrow 4-t=e^7 \rightarrow t=4-e^7$

c.) $\ln(t^2+t)=\ln 2 \rightarrow$

$e^{\ln(t^2+t)}=e^{\ln 2} \rightarrow t^2+t=2$

$\rightarrow t^2+t-2=0 \rightarrow (t-1)(t+2)=0 \rightarrow$

$\rightarrow$ $t=1$ or $t=-2$

d.) $\ln t + \ln(t+2) = 0 \rightarrow$

$\ln(t \cdot (t+2)) = 0 \rightarrow$

$e^{\ln(t^2+2t)} = e^0 \rightarrow t^2+2t=1 \rightarrow$

$t^2+2t-1=0 \rightarrow$

$$t = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2} \rightarrow$$

$$t = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2} \rightarrow$$

$t = -1+\sqrt{2}$ or $t = -1-\sqrt{2}$ (no! why?)

e.) $\ln(t+1) - \ln(t-1) = 1 \rightarrow$

$$\ln\left(\frac{t+1}{t-1}\right) = 1 \rightarrow \frac{t+1}{t-1} = e \rightarrow$$

$t+1 = et - e \rightarrow et - t = 1+e \rightarrow$

$(e-1)t = 1+e \rightarrow$

$$t = \frac{1+e}{e-1}$$

f.) $(\ln t)^2 - \ln t - 2 = 0 \rightarrow$

$(\ln t - 2)(\ln t + 1) = 0 \rightarrow$

$\ln t = 2 \rightarrow t = e^2$ or

$\ln t = -1 \rightarrow t = e^{-1}$

g.) $(\ln t)^3 - \ln t = 0 \rightarrow$

$\ln t\,((\ln t)^2 - 1) = 0 \rightarrow$

$\ln t\,(\ln t - 1)(\ln t + 1) = 0 \rightarrow$

$\ln t = -1 \rightarrow t = e^{-1}$

$\ln t = 1 \rightarrow t = e$

$\ln t = 0 \rightarrow t = 1$

h.) $\dfrac{\ln t}{3 + \ln t} = \dfrac{\ln t + 1}{2\ln t + 1} \rightarrow$

$2(\ln t)^2 + \ln t = (\ln t + 1)(3 + \ln t) \rightarrow$

$2(\ln t)^2 + \ln t = (\ln t)^2 + 4\ln t + 3 \rightarrow$

$(\ln t)^2 - 3\ln t - 3 = 0 \rightarrow$

$$\ln t = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{21}}{2} \rightarrow$$

$\ln t = \frac{1}{2}(3 + \sqrt{21}) \rightarrow t = e^{\frac{1}{2}(3+\sqrt{21})}$ or

$\ln t = \frac{1}{2}(3 - \sqrt{21}) \rightarrow t = e^{\frac{1}{2}(3-\sqrt{21})}$

$\boxed{SA2}$ average inflation rate $= \dfrac{\text{change in price}}{\text{change in time}}$

$$= \frac{2{,}400{,}000 - 1/4 \quad DM}{1922 - 1919 \quad yr.} = 799{,}999.92 \ DM/yr.$$

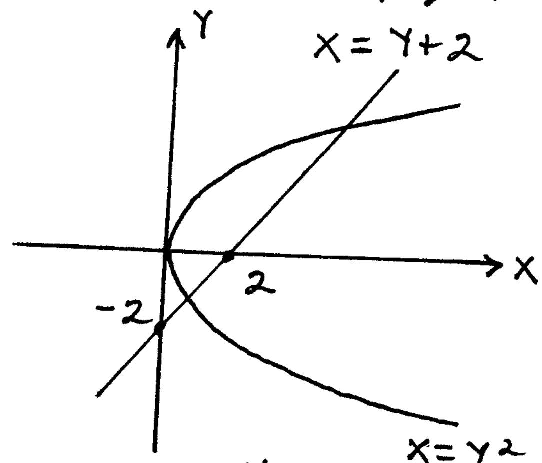
$\boxed{SA7}$ f.) $X = Y^2,\ X = Y + 2$

intersection :

$Y^2 = Y + 2 \rightarrow Y^2 - Y - 2 = 0 \rightarrow$

$(Y-2)(Y+1) = 0 \longrightarrow$

$Y = 2,\ X = 4$ and $Y = -1,\ X = 1$



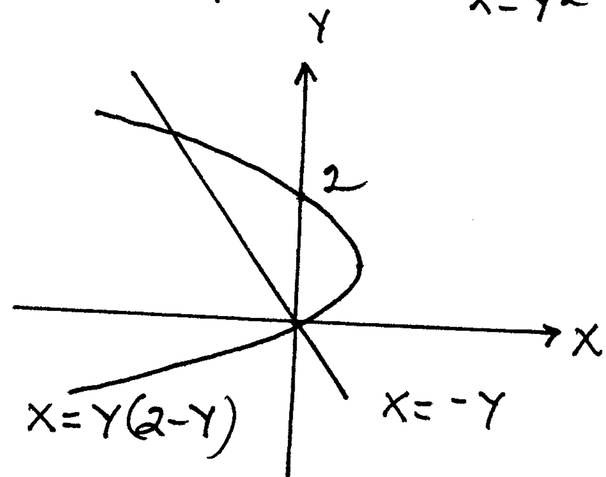
g.) $X = Y(2-Y),\ X = -Y$

intersection :

$2Y - Y^2 = -Y \rightarrow 0 = Y^2 - 3Y \rightarrow$

$0 = Y(Y-3) \longrightarrow$

$Y = 0,\ X = 0$ and $Y = 3,\ X = -3$



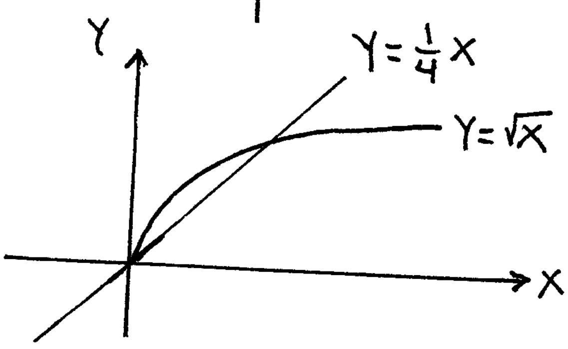
h.) $Y = \sqrt{X},\ Y = \frac{1}{4}X$

intersection :

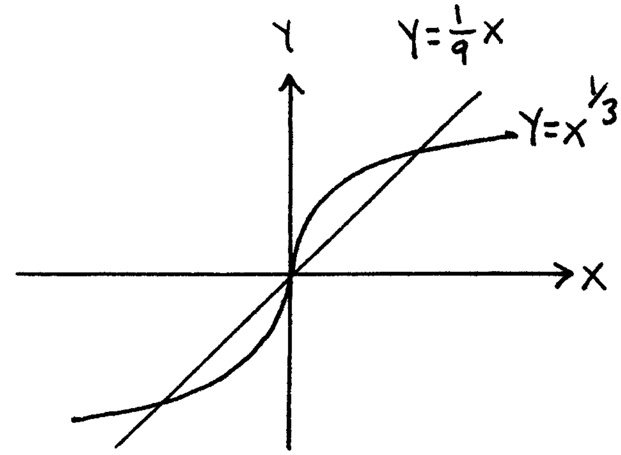
$\sqrt{X} = \frac{1}{4}X \rightarrow \sqrt{X} - \frac{1}{4}X = 0 \rightarrow$

$\sqrt{X}\left(1 - \frac{1}{4}\sqrt{X}\right) = 0 \rightarrow$

$X = 0,\ Y = 0$ and $X = 16,\ Y = 4$



i.) $Y = x^{1/3}$, $Y = \frac{1}{9}x$



intersection:

$x^{1/3} = \frac{1}{9}x \rightarrow x^{1/3} - \frac{1}{9}x = 0 \rightarrow$

$x^{1/3}\left(1 - \frac{1}{9}x^{2/3}\right) = 0 \rightarrow$

$x = 0, Y = 0$ and $x^{2/3} = 9 \rightarrow x^2 = 729 \rightarrow$

$x = 27, Y = 3$ and $x = -27, Y = -3$

$\boxed{5A17}$ c.) $100 = Ce^{k}$ and $200 = Ce^{4k} \rightarrow$

$C = \frac{100}{e^k} \rightarrow$ (substitute) $\rightarrow 200 = \left(\frac{100}{e^k}\right)e^{4k} \rightarrow$

$2 = e^{4k-k} \rightarrow 2 = e^{3k} \rightarrow \ln 2 = \ln e^{3k} \rightarrow$

$\ln 2 = 3k \rightarrow k = \frac{1}{3}\ln 2$ and $C = \frac{100}{e^k}$

$= \frac{100}{e^{\frac{1}{3}\ln 2}} = \frac{100}{e^{\ln 2^{1/3}}} = \frac{100}{2^{1/3}} = C \approx 79.37$

d.) $16 = Ce^{5k}$ and $90 = Ce^{20k} \rightarrow$

$C = \frac{16}{e^{5k}} \rightarrow$ (substitute) $\rightarrow 90 = \left(\frac{16}{e^{5k}}\right)e^{20k} \rightarrow$

$\frac{90}{16} = e^{20k-5k} \rightarrow \frac{45}{8} = e^{15k} \rightarrow \ln\left(\frac{45}{8}\right) = \ln e^{15k} \rightarrow$

$\ln\left(\frac{45}{8}\right) = 15k \rightarrow k = \frac{1}{15}\ln\left(\frac{45}{8}\right)$ and $C = \frac{16}{e^{5k}}$

$= \frac{16}{e^{5\left(\frac{1}{15}\ln\left(\frac{45}{8}\right)\right)}} = \frac{16}{e^{\frac{1}{3}\ln\left(\frac{45}{8}\right)}} = \frac{16}{e^{\ln\left(\frac{45}{8}\right)^{1/3}}}$

$\rightarrow C = \frac{16}{\left(\frac{45}{8}\right)^{1/3}} \approx 9.0$

e.) $50 = 7Ce^{3k+1}$ and $203 = 5Ce^{8k+11} \rightarrow$

$$C = \frac{50}{7e^{3k+1}} \rightarrow \text{(substitute)} \rightarrow$$

$$203 = 5\left(\frac{50}{7e^{3k+1}}\right)e^{8k+11} \rightarrow$$

$$\frac{1421}{250} = e^{8k+11-(3k+1)} = e^{5k+10} \rightarrow$$

$$\ln\left(\frac{1421}{250}\right) = \ln e^{5k+10} = 5k+10 \rightarrow$$

$$k = \frac{1}{5}\left[\ln\left(\frac{1421}{250}\right) - 10\right] \approx -1.652469 \;;$$

then $C = \dfrac{50}{7e^{3k+1}} \approx 373.725$