Section 4.5

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4.) $Y = \ln x^{1/2} = \frac{1}{2}\ln x \xrightarrow{D} y' = \frac{1}{2}\cdot\frac{1}{x}$ and
$x=1, Y=0$ so slope $m = Y' = \frac{1}{2}\cdot 1 = 1/2$

6.) $f(x) = \ln 2x \xrightarrow{D} f'(x) = \frac{1}{2x}\cdot 2 = \frac{1}{x}$

7.) $Y = \ln(x^2+3) \xrightarrow{D} Y' = \frac{1}{x^2+3}\cdot 2x$

10.) $Y = \ln(1-x)^{3/2} = \frac{3}{2}\ln(1-x) \xrightarrow{D}$
$y' = \frac{3}{2}\cdot\frac{1}{1-x}\cdot(-1)$

11.) $Y = \frac{1}{2}(\ln x)^6 \xrightarrow{D} Y' = \frac{1}{2}(6)(\ln x)^5\cdot\frac{1}{x}$

13.) $f(x) = x^2\ln x \xrightarrow{D}$
$f'(x) = x^2\cdot\frac{1}{x} + 2x\cdot\ln x$

14.) $y = \frac{\ln x}{x^2} \xrightarrow{D} y' = \frac{x^2\cdot\frac{1}{x} - \ln x\cdot 2x}{x^4}$

17.) $y = \ln\frac{x}{x+1} = \ln x - \ln(x+1) \xrightarrow{D}$
$y' = \frac{1}{x} - \frac{1}{x+1}$

20.) $y = \ln\sqrt{\frac{x+1}{x-1}} = \ln\left(\frac{x+1}{x-1}\right)^{1/2} = \ln\frac{(x+1)^{1/2}}{(x-1)^{1/2}}$
$= \ln(x+1)^{1/2} - \ln(x-1)^{1/2}$
$= \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1) \xrightarrow{D}$
$y' = \frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{2}\cdot\frac{1}{x-1}$

22.) $y = \ln(x \cdot \sqrt{4+x^2}) = \ln x + \ln(4+x^2)^{1/2}$

$= \ln x + \frac{1}{2}\ln(4+x^2) \xrightarrow{D}$

$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{4+x^2} \cdot 2x$

23.) $g(x) = e^{-x} \ln x \xrightarrow{D}$

$g'(x) = e^{-x} \cdot \frac{1}{x} + e^{-x} \cdot (-1) \cdot \ln x$

26.) $f(x) = \ln \frac{1+e^x}{1-e^x} = \ln(1+e^x) - \ln(1-e^x) \xrightarrow{D}$

$f'(x) = \frac{1}{1+e^x} \cdot e^x - \frac{1}{1-e^x} \cdot (-e^x)$

27.) $2^x = e^{\ln 2^x} = e^{x \cdot \ln 2}$

29.) $\log_4 x = \frac{\ln x}{\ln 4}$

31.) $\log_2 48 = \frac{\ln 48}{\ln 2} \approx 5.585$

34.) $\log_7 \frac{2}{9} = \frac{\ln(2/9)}{\ln 7} \approx -0.773$

37.) $y = 3^x \xrightarrow{D} y' = 3^x \cdot \ln 3$

40.) $g(x) = \log_5 x \xrightarrow{D} g'(x) = \frac{1}{x} \cdot \frac{1}{\ln 5}$

41.) $h(x) = 4^{2x-3} \xrightarrow{D} h'(x) = 4^{2x-3} \cdot (2) \cdot \ln 4$

43.) $y = \log_{10}(x^2+6x) \xrightarrow{D}$

$y' = \frac{1}{x^2+6x} \cdot (2x+6) \cdot \frac{1}{\ln 10}$

46.) $y = x \cdot 3^{x+1} \xrightarrow{D}$

$y' = x \cdot 3^{x+1} \cdot \ln 3 + (1) \cdot 3^{x+1}$

48.) $y = \frac{\ln x}{x} \xrightarrow{D} y' = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$,

and $x = e$, $y = \frac{1}{e}$ so slope

$m = \frac{1 - \ln e}{e^2} = \frac{1-1}{e^2} = \frac{0}{e^2} = 0$, and tangent

line is $y - \frac{1}{e} = (0)(x - e) = 0 \rightarrow y = \frac{1}{e}$

49.) $y = \log_3 (3x+7) \xrightarrow{D} y' = \frac{1}{3x+7} \cdot (3) \cdot \frac{1}{\ln 3}$,

and $x = \frac{2}{3}$, $y = 2$ so slope

$y' = m = \frac{1}{9} (3) \cdot \frac{1}{\ln 3} = \frac{1}{3 \ln 3}$, and tangent

line is $y - 2 = \frac{1}{3 \ln 3} \left(x - \frac{2}{3}\right)$

51.) $x^2 - 3 \ln y + y^2 = 10 \xrightarrow{D}$

$2x - 3 \cdot \frac{1}{y} y' + 2 y y' = 0 \longrightarrow$

$\left(2y - \frac{3}{y}\right) y' = -2x \longrightarrow y' = \frac{-2x}{2y - \frac{3}{y}}$

52.) $\ln (xy) + 5x = 30 \longrightarrow \ln x + \ln y + 5x = 30 \xrightarrow{D}$

$\frac{1}{x} + \frac{1}{y} y' + 5 = 0 \longrightarrow \frac{1}{y} y' = -5 - \frac{1}{x} \longrightarrow$

$y' = y \left(-5 - \frac{1}{x}\right)$

60.) $T = 87.97 + 34.96 \ln p + (7.91) p^{1/2} \xrightarrow{D}$

$\dfrac{dT}{dp} = 34.96 \left(\dfrac{1}{p}\right) + (7.91) \cdot \dfrac{1}{2} p^{-1/2}$ and $p = 60 \rightarrow$

$\dfrac{dT}{dp} = 34.96 \left(\dfrac{1}{60}\right) + \dfrac{7.91}{2} (60)^{-1/2} \approx 1.093 \ \dfrac{^\circ F}{lbs./in.^2}$

61.) $f(x) = 1 + 2x \ln x \xrightarrow{D} f'(x) = 2x \cdot \dfrac{1}{x} + 2 \ln x$, and

$x = 1, y = 1$ so slope $m = f'(1) = 2 + 2 \cancel{\ln 1}^{0} = 2$,

and tangent line is $y - 1 = 2(x - 1) \rightarrow$

$y = 2x - 1$

66.) $f(x) = x^2 \log_3 x \xrightarrow{D}$

$f'(x) = x^2 \cdot \dfrac{1}{x} \cdot \dfrac{1}{\ln 3} + 2x \cdot \log_3 x$, and

$x = 1, y = 0$ so slope $m = f'(1) = \dfrac{1}{\ln 3} + 2 \cancel{\log_3 1}^{0} = \dfrac{1}{\ln 3}$,

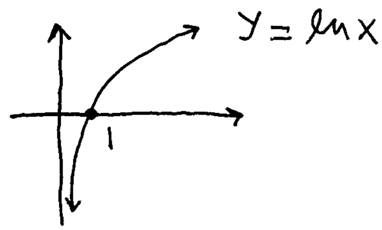
so tangent line is $y - 0 = \dfrac{1}{\ln 3}(x - 1)$

67.) $y = x - \ln x$, Domain : $x > 0$

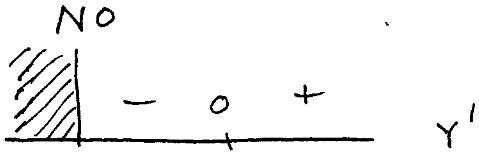
$\xrightarrow{D} y' = 1 - \dfrac{1}{x} = \dfrac{x-1}{x} = 0 \rightarrow$

$x - 1 = 0 \rightarrow x = 1$;

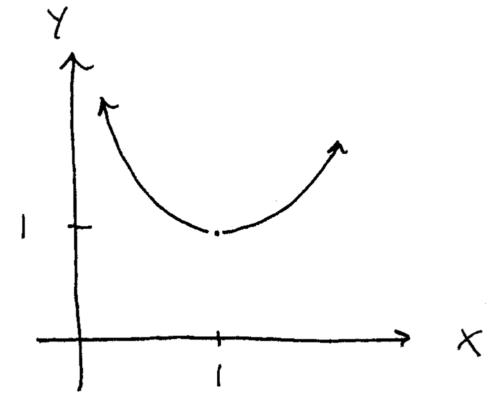
$\lim\limits_{x \to 0^+} (x - \ln x) = 0 - (-\infty) = \infty$, so



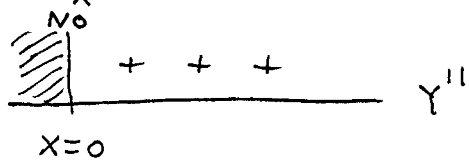
H.A. is $x = 0$;



$\left.\begin{array}{l} X=1 \\ Y=1 \end{array}\right\}$ abs. min.



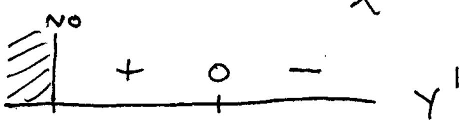
$Y'' = \dfrac{1}{x^2}$



Y is ↑ for $x>1$,
Y is ↓ for $0<x<1$,
Y is ∪ for $x>0$

Domain : $x>0$

69.) $Y = \dfrac{\ln x}{x} \rightarrow Y' = \dfrac{x \cdot \frac{1}{x} - \ln x}{x^2} = \dfrac{1-\ln x}{x^2} = 0$



$\left.\begin{array}{l} x=e \\ Y=\frac{1}{e} \end{array}\right\}$ abs. max.

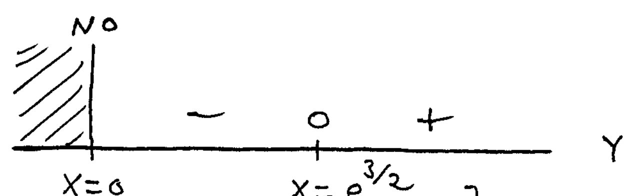
$Y'' = \dfrac{x^2\left(\frac{-1}{x}\right) - (1-\ln x)(2x)}{x^4}$

$= \dfrac{-x-2x+2x\ln x}{x^4}$

$= \dfrac{-3x+2x\ln x}{x^4}$

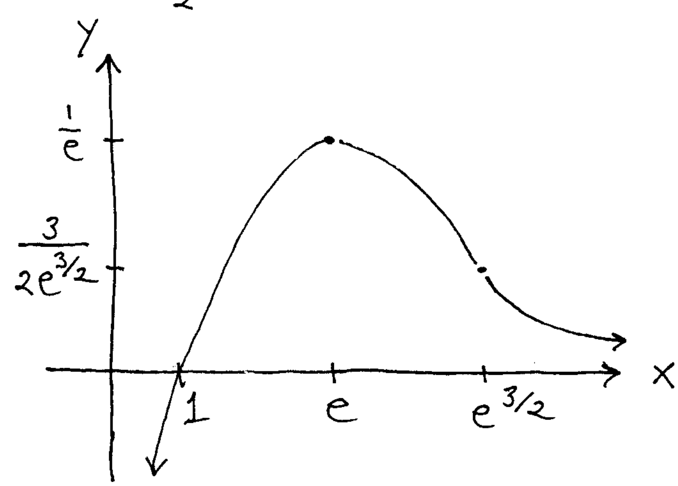
$= \dfrac{x(-3+2\ln x)}{x^4}$

$= \dfrac{-3+2\ln x}{x^3} = 0 \rightarrow \ln x = \dfrac{3}{2} \rightarrow x = e^{3/2}$



$\left.\begin{array}{l} x=e^{3/2} \\ Y=\dfrac{3/2}{e^{3/2}} \end{array}\right\}$ infl. pt.

$Y=0 \rightarrow x=1$
Y is ↑ for $0<x<e$,
Y is ↓ for $x>e$,
Y is ∪ for $x>e^{3/2}$,
Y is ∩ for $0<x<e^{3/2}$



72.)

$$Y = (\ln x)^2 \rightarrow Y' = 2\ln x \cdot \frac{1}{x} = 0$$

Domain : $x > 0$

NO

Y': $-$ $\;0\;$ $+$

$x=0$ $\quad$ $\left.\begin{matrix} x=1 \\ Y=0 \end{matrix}\right\}$ abs. min.

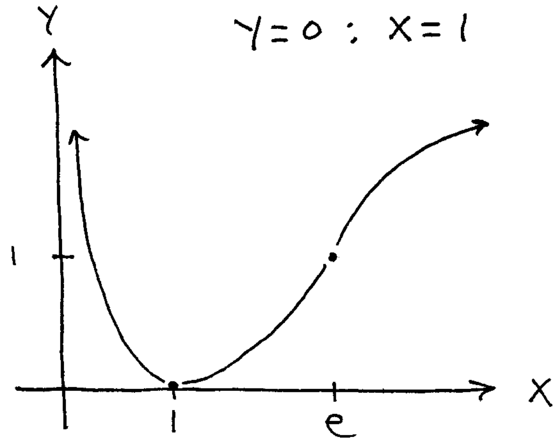
$$Y'' = \frac{x \cdot \frac{2}{x} - 2\ln x}{x^2} = \frac{2(1-\ln x)}{x^2} = 0$$

NO

Y'': $+$ $\;0\;$ $-$

$x=0$ $\quad$ $\left.\begin{matrix} x=e \\ Y=1 \end{matrix}\right\}$ infl. pt.

$Y=0$: $x=1$



Y is $\uparrow$ for $x > 1$,
Y is $\downarrow$ for $0 < x < 1$,
Y is $\cup$ for $0 < x < e$,
Y is $\cap$ for $x > e$

82.) $$t = \frac{5.315}{-6.7968 + \ln x}$$

b.) $x = 1167.41 \rightarrow t \approx 20$ yrs. ;

total paid : $(1167.41)(20)(12) \approx \$280{,}185.89$

c.) $x = 1068.45 \rightarrow t \approx 30$ yrs. ;

total paid : $(1068.45)(30)(12) \approx \$384{,}647.18$

d.) $t = 5.315(-6.7968 + \ln x)^{-1} \xrightarrow{D}$

$$\frac{dt}{dx} = -5.315(-6.7968 + \ln x)^{-2} \cdot \frac{1}{x} \longrightarrow$$

i.) $x = 1167.41 \rightarrow \frac{dt}{dx} \approx -0.064$ yrs./\$

ii.) $x = 1068.45 \rightarrow \frac{dt}{dx} \approx -0.158$ yrs./\$

Handout 3

1.) a.) $Y' = \frac{1}{\sin x} \cdot \cos x - \sin(\ln x) \cdot \frac{1}{x}$

b.) $Y' = 10 \cdot \frac{1}{3\tan x + e^{5x}} \cdot (3\sec^2 x + 5e^{5x})$

c.) $f'(x) = \csc x \cdot \frac{1}{\cos^5 4x} \cdot 5\cos^4 4x \cdot {}^-\sin 4x \cdot 4$

$- \csc x \cot x \cdot \ln(\cos^5 4x)$

d.) $Y = \ln\left(\frac{x \sec x}{\cot x + \sin x}\right)$

$= \ln x + \ln(\sec x) - \ln(\cot x + \sin x) \longrightarrow$

$Y' = \frac{1}{x} + \frac{\sec x \tan x}{\sec x} - \frac{-\csc^2 x + \cos x}{\cot x + \sin x}$

e.) $Y = 100\left[\ln(1+e^{-6x})^{1/3}\right]^{-5} \longrightarrow$

$Y' = -500\left[\ln(1+e^{-6x})^{\frac{1}{3}}\right]^{-6} \cdot \frac{1}{(1+e^{-6x})^{1/3}} \cdot \frac{1}{3}(1+e^{-6x})^{-\frac{2}{3}} \cdot -6e^{-6x}$

f.) $Y' = \frac{1}{\ln(\ln(\sqrt{7-x}))} \cdot \frac{1}{\ln(\sqrt{7-x})} \cdot \frac{1}{\sqrt{7-x}} \cdot \frac{1}{2}(7-x)^{-1/2} \cdot (-1)$

2.) $Y = \frac{2+\ln x}{1+x\ln x}$ at $x=1 \rightarrow Y=2$ and

$Y' = \frac{(1+x\ln x)(\frac{1}{x}) - (2+\ln x)(x \cdot \frac{1}{x} + \ln x)}{(1+x\ln x)^2}$ at $x=1 \rightarrow$

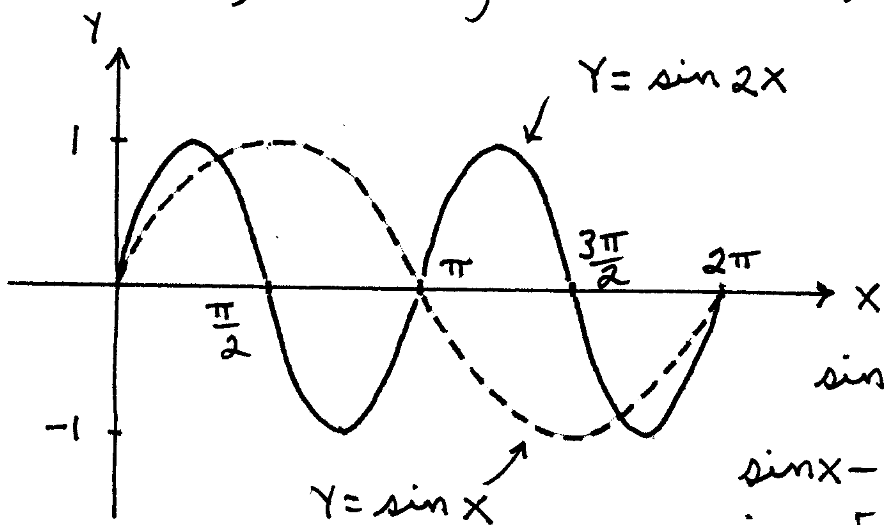
$Y' = \frac{1-2}{1^2} = -1$ so tangent line is $Y-2 = -(x-1)$ or $Y = {}^-x+3$

3.) $Y = \ln\left(\frac{x}{x+1}\right) = \ln x - \ln(x+1) \rightarrow Y' = \frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)}$;

$2Y = x-5 \rightarrow Y = \frac{1}{2}x - \frac{5}{2}$ so slope $\frac{1}{2} = \frac{1}{x(x+1)} \rightarrow x(x+1) = 2 \rightarrow$

$x^2+x-2=0 \rightarrow (x-1)(x+2)=0 \rightarrow x=1, Y=\ln\frac{1}{2}$ and $x={}^-2, Y=\ln 2$

$\boxed{SA7}$ j.) $Y = \sin X$, $Y = \sin 2X$ on $[0, 2\pi]$



intersection:

$\sin X = \sin 2X \rightarrow$

$\sin X = 2 \sin X \cos X \rightarrow$

$\sin X - 2 \sin X \cos X = 0 \rightarrow$

$\sin X [1 - 2 \cos X] = 0 \rightarrow$

$\sin X = 0$ and $\cos X = 1/2$

$\downarrow$

$X = 0$, $Y = 0$

$X = \pi$, $Y = 0$

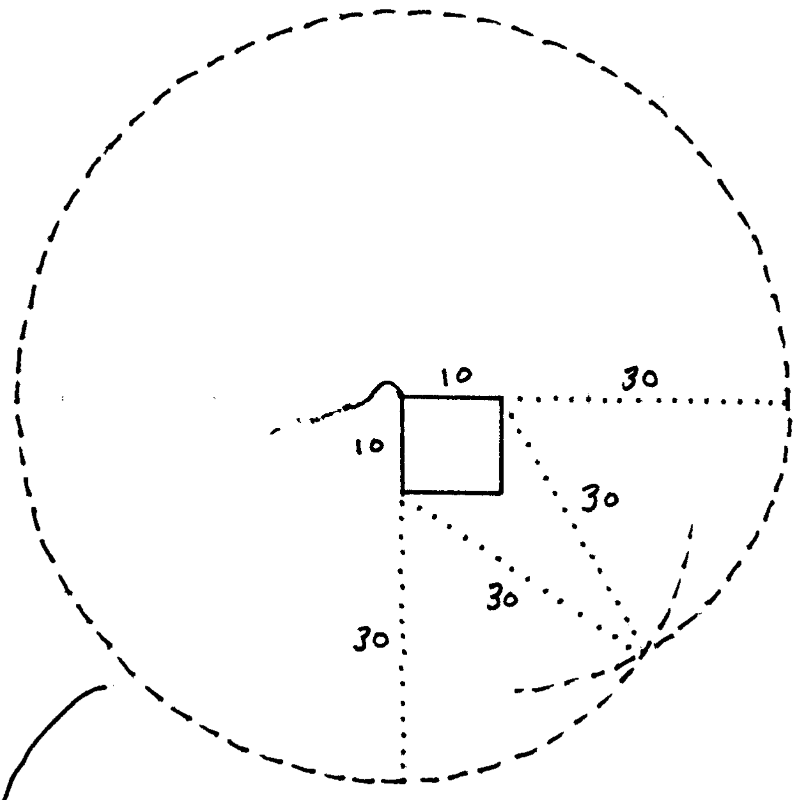
$X = 2\pi$, $Y = 0$

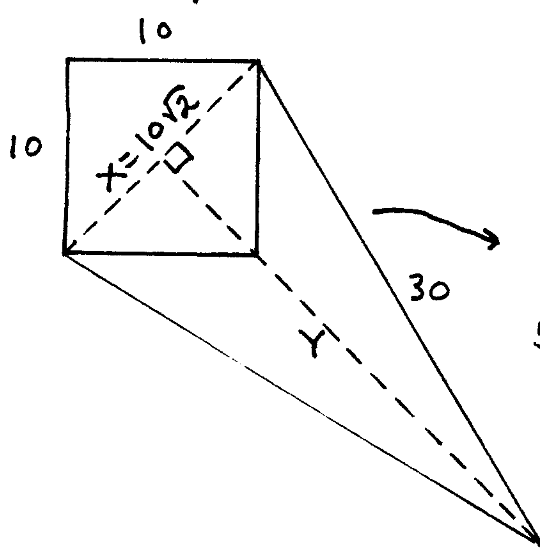
$\downarrow$

$X = \frac{\pi}{3}$, $Y = \frac{\sqrt{3}}{2}$

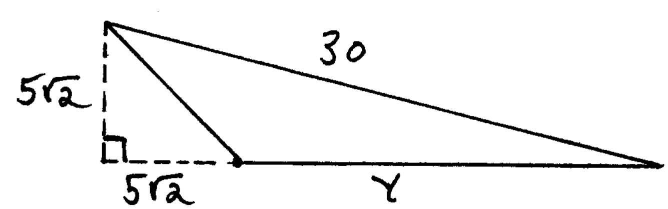
$X = \frac{5\pi}{3}$, $Y = -\frac{\sqrt{3}}{2}$

$\boxed{SA14}$





$10^2 + 10^2 = X^2 \rightarrow X^2 = 200 \rightarrow$
$X = 10\sqrt{2}$



$(5\sqrt{2})^2 + (5\sqrt{2} + Y)^2 = 30^2 \rightarrow$

$50 + (5\sqrt{2} + Y)^2 = 900 \rightarrow$

$(5\sqrt{2} + Y)^2 = 850 \rightarrow 5\sqrt{2} + Y = 5\sqrt{34} \rightarrow$

closest distance to shed is

$Y = 5\sqrt{34} - 5\sqrt{2} \approx 22.08$ ft.