Section 4.6

1.) $Y = Ce^{kt}$ and $t=0, Y=2$ so
$2 = Ce^0 \to \underline{C=2} \to Y = 2e^{kt}$ and $t=4, Y=3 \to$
$3 = 2e^{4k} \to \frac{3}{2} = e^{4k} \to \ln\frac{3}{2} = \ln e^{4k} \to$
$\ln\frac{3}{2} = 4k \to \underline{k = \frac{1}{4}\ln\frac{3}{2}}$ so $\boxed{Y = 2e^{\left(\frac{1}{4}\ln\frac{3}{2}\right)t}}$

9.) $\frac{dY}{dt} = -4Y$ so $Y = ce^{-4t}$ and $t=0, Y=30$
so $30 = ce^0 \to c = 30$ and $\boxed{Y = 30e^{-4t}}$,
exponential decay

11.) $A = Ce^{kt}$ and $t = 0$ yr., $A = 10$ gm. so
$10 = Ce^0 \to \underline{C=10} \to A = 10e^{kt}$; when $t = 1599$ yrs.,
$A = 5$ gm. so $5 = 10e^{1599k} \to \frac{1}{2} = e^{1599k} \to$
$\ln\frac{1}{2} = \ln e^{1599k} \to \ln\frac{1}{2} = 1599k \to \underline{k = \frac{1}{1599}\ln\frac{1}{2}} \to$

$$A = 10 \cdot e^{\left(\frac{1}{1599}\ln\frac{1}{2}\right)t} ;$$

if $t = 1000$ yrs., then $A = 6.482$ gm. ;
if $t = 10{,}000$ yrs., then $A = 0.131$ gm.

18.) $A = Ce^{kt}$ and $t=0 \to A = Ce^0 = C$ is initial amount ; if $t = 1$ yr., $A = 0.9957C$ so
$0.9957\cancel{C} = \cancel{C}e^{k(1)} \to \ln 0.9957 = \ln e^k = k$ so
$A = Ce^{(\ln 0.9957)t}$; if $A = \frac{1}{2}C$ then $\frac{1}{2}\cancel{C} = \cancel{C}e^{(\ln 0.9957)t}$
$\to \ln\frac{1}{2} = \ln e^{(\ln 0.9957)t} \to \ln\frac{1}{2} = (\ln 0.9957)t \to$

$$t = \frac{\ln \frac{1}{2}}{\ln 0.9957} \approx \boxed{160.85 \text{ yrs.}}$$

19.) $A = Ce^{kt}$ and C is initial amount ; if $t = 5715$ yrs., $A = \frac{1}{2}C \rightarrow \frac{1}{2}\cancel{C} = \cancel{C}e^{k(5715)} \rightarrow$ $\ln \frac{1}{2} = \ln e^{k(5715)} \rightarrow \ln \frac{1}{2} = 5715k \rightarrow k = \frac{1}{5715} \ln \frac{1}{2} \rightarrow$ $A = Ce^{\left(\frac{1}{5715}\ln\frac{1}{2}\right)t}$; if $A = 0.15C$ then $0.15\cancel{C} = \cancel{C}\, e^{\left(\frac{1}{5715}\ln\frac{1}{2}\right)t} \rightarrow \ln 0.15 = \ln e^{\left(\frac{1}{5715}\ln\frac{1}{2}\right)t} \rightarrow$ $\ln 0.15 = \left(\frac{1}{5715} \ln \frac{1}{2}\right)t \rightarrow \quad t = \frac{5715 \ln 0.15}{\ln 1/2} \approx 15{,}641.8$ yrs.

21.) $A = Ce^{kt}$ and $t = 0$ hr., $A = 150$ so $\underline{C = 150}$ $\rightarrow A = 150e^{kt}$; if $t = 5$ hrs., $A = 450 \rightarrow$ $450 = 150e^{5k} \rightarrow 3 = e^{5k} \rightarrow \ln 3 = \ln e^{5k} \rightarrow$ $\ln 3 = 5k \rightarrow k = \frac{1}{5}\ln 3 \rightarrow$

$$A = 150\, e^{\left(\frac{1}{5}\ln 3\right)t} \quad ;$$

a.) if $t = 10$ hrs., then $A = 1350$ bacteria

b.) if $A = 300$ bacteria, then

$300 = 150\, e^{\left(\frac{1}{5}\ln 3\right)t} \rightarrow 2 = e^{\left(\frac{1}{5}\ln 3\right)t} \rightarrow$

$\ln 2 = \ln e^{\left(\frac{1}{5}\ln 3\right)t} \rightarrow \ln 2 = \left(\frac{1}{5}\ln 3\right)t \rightarrow$

$t = \frac{5 \ln 2}{\ln 3} \approx 3.15$ hrs.

40.) $S = 30(1 - 3^{kt})$

a.) if $t = 1$ yr., $S = 5$ (thousands) $\longrightarrow$

$5 = 30(1 - 3^{k}) \rightarrow \frac{1}{6} = 1 - 3^{k} \rightarrow 3^{k} = \frac{5}{6} \longrightarrow$

$\ln 3^k = \ln \frac{5}{6} \rightarrow k \ln 3 = \ln \frac{5}{6} \rightarrow k = \frac{\ln \frac{5}{6}}{\ln 3} \approx -0.166$

b.) saturation:

$\lim_{t\to\infty} S = \lim_{t\to\infty} 30\left(1 - 3^{-0.166t}\right) = 30(1-0) = 30$

so 30,000 units will saturate market

c.) if $t = 5$ yrs., then $S = 30\left(1 - 3^{-0.166(5)}\right) \approx 17.944$

or 17,944 units

d.) $S = 30\left(1 - 3^{-0.166t}\right) \rightarrow$

$S' = 30(0.166)\, 3^{-0.166t} \ln 3 > 0$ and

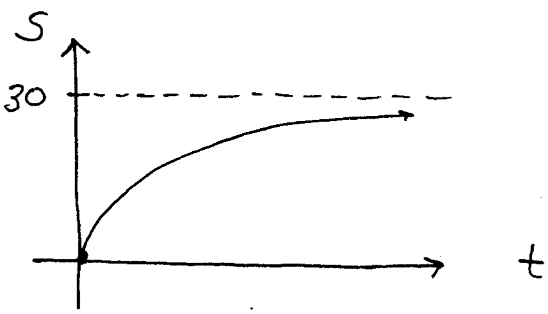
$S'' = -30(0.166)^2\, 3^{-0.166t} (\ln 3)^2 < 0$ so

S is $\uparrow$,

S is $\cap$,

$S = 0$ if $t = 0$,

$\lim_{t\to\infty} S = 30$



44.) a.) $p = Ce^{kx}$ and $p = \$45$, $X = 1000$ units $\rightarrow$

$45 = Ce^{1000k}$ and $p = \$40$, $X = 1200$ units $\rightarrow$

$40 = Ce^{1200k}$; solve for C in one and substitute into the other: $C = \frac{45}{e^{1000k}} \rightarrow$ (sub.)

$40 = \left(\frac{45}{e^{1000k}}\right) e^{1200k} \rightarrow \frac{8}{9} = e^{200k} \rightarrow \ln\left(\frac{8}{9}\right) = \ln\left(e^{200k}\right)$

$\rightarrow \ln\left(\frac{8}{9}\right) = 200k \rightarrow k = \frac{\ln(8/9)}{200} \approx -0.0005889$

and $C = \frac{45}{e^{1000k}} \approx 81.1$ so $p = 81.1\, e^{-0.0005889X}$;

b.) Maximize revenue $R = xp \rightarrow$

$$R = x\left(81.1\, e^{-0.0005889x}\right) = 81.1x\, e^{-0.0005889x} \xrightarrow{D}$$

$$R' = (81.1x)(-0.0005889)e^{-0.0005889x} + 81.1\, e^{-0.0005889x}$$

$$= 81.1\, e^{-0.0005889x}\left[1 - 0.0005889x\right] = 0$$

$\rightarrow x \approx 1698$

$$\begin{array}{ccccc} + & & 0 & & - \\ \hline \end{array} \quad R'$$

and max. rev. $x = 1698$ units, $p = \$29.84$, $R = \$50{,}659.87$

46.) $V = 100{,}000\, e^{0.75\sqrt{t}}$ and maximize

$$A = Ve^{-0.04t} = 100{,}000\, e^{0.75\sqrt{t}}e^{-0.04t} = 100{,}000\, e^{0.75\sqrt{t} - 0.04t} \xrightarrow{D}$$

$$A' = 100{,}000\, e^{0.75\sqrt{t} - 0.04t}\left(0.75 \tfrac{1}{2} t^{-1/2} - 0.04\right) = 0 \rightarrow$$

$$\frac{0.75}{2\sqrt{t}} = 0.04 \rightarrow \sqrt{t} = \frac{0.75}{0.08} \rightarrow t = \left(\frac{0.75}{0.08}\right)^2 \rightarrow$$

$t \approx 88$ years

$$\begin{array}{ccccc} + & & 0 & & - \\ \hline \end{array} \quad A'$$

and max. pres. value $t = 88$ yrs (2078), $A = \$3{,}363{,}694$

48.) $R = \frac{\ln I - \cancel{\ln 1}^{\,0}}{\ln 10} = \frac{\ln I}{\ln 10}$

a.) $8.3 = \frac{\ln I}{\ln 10} \rightarrow 8.3 \ln 10 = \ln I \rightarrow \ln 10^{8.3} = \ln I \rightarrow$

$I = 10^{8.3}$

b.) $16.6 = \frac{\ln I}{\ln 10} \rightarrow 16.6 \ln 10 = \ln I \rightarrow \ln 10^{16.6} = \ln I \rightarrow$

$I = 10^{16.6}$, then intensity is increased by a factor of $\frac{10^{16.6}}{10^{8.3}} = 10^{8.3} = 199,526,232$

c.) $R = \frac{1}{\ln 10} \ln I \rightarrow \frac{dR}{dI} = \frac{1}{\ln 10} \cdot \frac{1}{I}$

$\boxed{\text{SA3}}$: $T = c \cdot S^{3/2}$ and for earth

$T = 365$ days, $S = 150$ million km $\rightarrow$

$365 = c \cdot 150^{3/2} \rightarrow c = \frac{365}{150^{3/2}}$ so

$$\boxed{T = \frac{365}{150^{3/2}} \cdot S^{3/2}}$$

a.) Mercury : $S = 58$ million km $\rightarrow$

period $T = 87.76$ days.

b.) Pluto : $S = 6000$ million km $\rightarrow$

period $T = 92,338.51$ days

$= 252.98$ years