# Section 5.1

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2.) $D\left(8\sqrt{x}+c\right) = 8\cdot\frac{1}{2}x^{-1/2}+0 = \frac{4}{\sqrt{x}}$

so $\int \frac{4}{\sqrt{x}}\,dx = 8\sqrt{x}+c$

5.) $D\left(\frac{1}{5}\cdot 4x^{3/2}(x-5)+c\right) = D\left(\frac{4}{5}\left(x^{5/2}-5x^{3/2}\right)+c\right)$

$= \frac{4}{5}\cdot\left(\frac{5}{2}x^{3/2}-\frac{15}{2}x^{1/2}\right)+0 = \frac{20}{10}x^{3/2}-\frac{60}{10}x^{1/2}$

$= 2x^{3/2}-6x^{1/2} = 2\sqrt{x}\,(x-3)$ so

$\int 2\sqrt{x}\,(x-3)\,dx = \frac{4}{5}x^{3/2}(x-5)+c$

9.) $\int 6\,dx = 6x+c$

12.) $\int 3t^4\,dt = 3\cdot\frac{1}{5}t^5+c$

13.) $\int 5x^{-3}\,dx = 5\cdot\frac{x^{-2}}{-2}+c$

15.) $\int du = \int 1\,du = u+c$

18.) $\int e^3\,dy = e^3y+c$

22.) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1}+c$

24.) $\int \frac{1}{x^2x^{1/2}}\,dx = \int \frac{1}{x^{5/2}}\,dx = \int x^{-5/2}\,dx$

$= \frac{x^{-3/2}}{-3/2}+c$

25.) $\int x(x^2+3)\,dx = \int (x^3+3x)\,dx = \frac{x^4}{4}+3\cdot\frac{x^2}{2}+c$

34.) $\int (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - x^2 + 3x + C$

36.) $\int \left(x^{1/2} + \frac{1}{2x^{1/2}}\right) dx = \int \left(x^{1/2} + \frac{1}{2} \cdot x^{-1/2}\right) dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C$

41.) $\int \frac{2x^3 + 1}{x^3}\,dx = \int \left(\frac{2x^3}{x^3} + \frac{1}{x^3}\right) dx$

$= \int (2 + x^{-3})\,dx = 2x + \frac{x^{-2}}{-2} + C$

44.) $\int x^{1/2}(x+1)\,dx = \int (x^{3/2} + x^{1/2})\,dx = \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + C$

45.) $\int (x-1)(6x-5)\,dx = \int (6x^2 - 11x + 5)\,dx$

$= 6 \cdot \frac{x^3}{3} - 11 \cdot \frac{x^2}{2} + 5x + C$

49.) $f'(x) = 3x^{1/2} + 3 \rightarrow f(x) = \int (3x^{1/2} + 3)\,dx$

$= 3 \cdot \frac{x^{3/2}}{3/2} + 3x + C = \cancel{3} \cdot \frac{2}{\cancel{3}} x^{3/2} + 3x + C \rightarrow$

$f(x) = 2x^{3/2} + 3x + C$ and $x = 1, y = 4 \rightarrow$

$4 = 2(1)^{3/2} + 3(1) + C \rightarrow 4 = 5 + C \rightarrow C = -1 \rightarrow$

$f(x) = 2x^{3/2} + 3x - 1$

51.) $f'(x) = 6x(x-1) = 6x^2 - 6x \rightarrow$

$f(x) = \int (6x^2 - 6x)\,dx = 2x^3 - 3x^2 + C \rightarrow$

$f(x) = 2x^3 - 3x^2 + C$ and $x = 1, y = -1 \rightarrow$

$-1 = 2(1)^3 - 3(1)^2 + c \rightarrow -1 = -1 + c \rightarrow c = 0 \rightarrow$

$f(x) = 2x^3 - 3x^2$

54.) $f'(x) = \dfrac{x^2 - 5}{x^2} = \dfrac{x^2}{x^2} - \dfrac{5}{x^2} = 1 - 5x^{-2} \rightarrow$

$f(x) = \int (1 - 5x^{-2})\,dx = x - 5 \cdot \dfrac{x^{-1}}{-1} + c \rightarrow$

$f(x) = x + \dfrac{5}{x} + c$ and $x = 1,\ y = 2 \rightarrow$

$2 = 1 + \dfrac{5}{1} + c \rightarrow 2 = 6 + c \rightarrow c = -4 \rightarrow$

$f(x) = x + \dfrac{5}{x} - 4$

56.) $y' = 2(x-1) = 2x - 2 \rightarrow$

$y = \int (2x - 2)\,dx = x^2 - 2x + C \rightarrow y = x^2 - 2x + c$

and $x = 3,\ y = 2 \rightarrow 2 = 3^2 - 2(3) + c \rightarrow$

$2 = 9 - 6 + c \rightarrow 2 = 3 + c \rightarrow c = -1 \rightarrow$

$y = x^2 - 2x - 1$

57.) $f'(x) = 6x^{1/2} - 10 \rightarrow f(x) = \int (6x^{1/2} - 10)\,dx$

$= 6 \cdot \dfrac{x^{3/2}}{3/2} - 10x + c = 6 \cdot \dfrac{2}{3} x^{3/2} - 10x + c \rightarrow$

$f(x) = 4x^{3/2} - 10x + c$ and $x = 4,\ y = 2 \rightarrow$

$2 = 4 \cdot 4^{3/2} - 10(4) + c \rightarrow 2 = 4 \cdot (8) - 40 + c \rightarrow$

$2 = -8 + c \rightarrow c = 10 \rightarrow$

$f(x) = 4x^{3/2} - 10x + 10$

59.) $\underline{f''(x) = 2} \rightarrow f'(x) = 2x + c$ and
$x = 2,\ f'(2) = 5 \rightarrow 5 = 2(2) + c \rightarrow c = 1$
$\rightarrow \underline{f'(x) = 2x + 1} \rightarrow f(x) = x^2 + x + c$
and $x = 2,\ f(2) = 10 \rightarrow 10 = 2^2 + 2 + c \rightarrow$
$10 = 6 + c \rightarrow c = 4 \rightarrow \underline{f(x) = x^2 + x + 4}$

60.) $\underline{f''(x) = x^2} \rightarrow f'(x) = \frac{1}{3}x^3 + c$ and
$x = 0,\ f'(0) = 6 \rightarrow 6 = \frac{1}{3}(0)^3 + c \rightarrow c = 6 \rightarrow$
$\underline{f'(x) = \frac{1}{3}x^3 + 6} \rightarrow f(x) = \frac{1}{3} \cdot \frac{1}{4}x^4 + 6x + c \rightarrow$
$f(x) = \frac{1}{12}x^4 + 6x + c$ and $x = 0,\ f(0) = 3 \rightarrow$
$3 = \frac{1}{12}(0)^4 + 6(0) + c \rightarrow c = 3 \rightarrow$
$f(x) = \frac{1}{12}x^4 + 6x + 3$

75.) <u>acceleration</u>: $a(t) = -32$ ft./sec.$^2$ $\rightarrow$

<u>velocity</u>: $v(t) = -32t + C$ and $v(0) = 60$ ft./sec.

$\rightarrow 60 = 0 + C \rightarrow C = 60 \rightarrow \boxed{v(t) = -32t + 60} \rightarrow$

<u>height</u>: $s(t) = -16t^2 + 60t + C$ and $s(0) = 0$ ft. $\rightarrow$

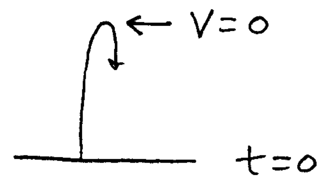
$0 = 0 + C \rightarrow C = 0 \rightarrow \boxed{s(t) = -16t^2 + 60t}$ ;

highest point $\rightarrow v(t) = 0 \rightarrow$

$0 = -32t + 60 \rightarrow t = \frac{60}{32} = \frac{15}{8}$ sec.

and $s\left(\frac{15}{8}\right) = -16\left(\frac{225}{64}\right) + 60\left(\frac{15}{8}\right)$

$= \frac{225}{4} = 56.25$ ft.




76.) <u>acceleration</u>: $a(t) = -32$ ft./sec.$^2$ $\rightarrow$

<u>velocity</u>: $v(t) = -32t + C$ and $v(0) = 0$ ft./sec.

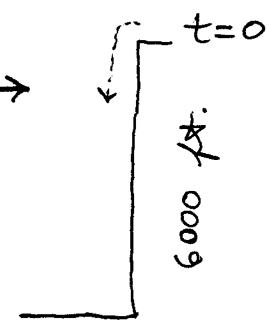
$\rightarrow 0 = 0 + C \rightarrow C = 0 \rightarrow v(t) = -32t \rightarrow$

<u>height</u>: $s(t) = -16t^2 + C$ and $s(0) = 6000 \rightarrow$

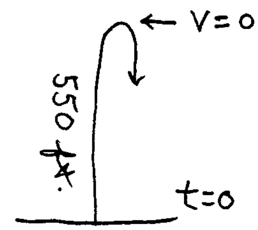
$6000 = 0 + C \rightarrow C = 6000 \rightarrow s(t) = -16t^2 + 6000$ ;

hit floor $\rightarrow s(t) = 0 \rightarrow 0 = -16t^2 + 6000 \rightarrow$

$t^2 = \frac{6000}{16} \rightarrow t \approx 19.36$ sec.



77.) Let $A$ be initial velocity so $v(0) = A$ ft./sec. :




<u>acceleration</u>: $a(t) = -32$ ft./sec.$^2$ $\rightarrow$

<u>velocity</u>: $v(t) = -32t + c$ and $v(0) = A \rightarrow$

$A = 0 + c \rightarrow c = A \rightarrow v(t) = -32t + A \rightarrow$

<u>height</u>: $s(t) = -16t^2 + At + c$ and $s(0) = 0 \rightarrow$

$0 = 0 + c \rightarrow c = 0 \rightarrow s(t) = -16t^2 + At$ ;

at highest point $v(t) = 0 \rightarrow 0 = -32t + A \rightarrow$

$t = \frac{A}{32}$ sec. ; if highest point is 550 ft.

then $s\left(\frac{A}{32}\right) = 550 \rightarrow -16\left(\frac{A}{32}\right)^2 + A\left(\frac{A}{32}\right) = 550 \rightarrow$

$\frac{A^2}{64} = 550 \rightarrow A^2 = 35{,}200 \rightarrow A \approx 187.62$ ft./sec.

78.) <u>acceleration</u>: $a(t) = -32$ ft./sec.$^2$ $\rightarrow$

<u>velocity</u>: $v(t) = -32t + c$ and $v(0) = 16$ ft./sec. $\rightarrow$

$16 = 0 + c \rightarrow c = 16 \rightarrow \boxed{v(t) = -32t + 16} \rightarrow$

<u>height</u>: $s(t) = -16t^2 + 16t + c$ and $s(0) = 64$ ft. $\rightarrow$

$64 = 0 + c \rightarrow c = 64 \rightarrow s(t) = -16t^2 + 16t + 64$ ;

a.) hit ground : $s(t) = 0 \rightarrow -16t^2 + 16t + 64 = 0 \rightarrow$

$-16(t^2 - t - 4) = 0 \rightarrow t = \frac{1 \pm \sqrt{1+16}}{2}$

$\rightarrow t = \frac{1}{2}(1 + \sqrt{17})$ sec. $\approx 2.56$ sec.

b.) strike velocity :

$v\left(\frac{1}{2}(1+\sqrt{17})\right) = -32\left(\frac{1}{2}(1+\sqrt{17})\right) + 16 = -16\sqrt{17}$ ft./sec. $\approx -66$ ft./sec.

Handout 4

1.) $D\left(\frac{1}{2}\sin 2x + 30\right) = \frac{1}{2}\cdot\cos 2x \cdot 2 = \cos 2x$

2.) $D\left(\frac{1}{8}\tan 8x + 4\sec 5x - 3\right) = \frac{1}{8}\sec^2 8x \cdot 8 + 4\sec 5x \tan 5x \cdot 5$
$= \sec^2 8x + 20\sec 5x \tan 5x$

3.) $\int \sin 9x \, dx = \frac{-1}{9}\cos 9x + c$

check: $D\left(\frac{-1}{9}\cos 9x + c\right) = \frac{-1}{9}\cdot -\sin 9x \cdot 9 = \sin 9x$

4.) $\int \sin kx \, dx = \frac{-1}{k}\cos kx + c$

5.) $D(e^x \cos x) = e^x \cdot -\sin x + e^x \cos x = e^x \cos x - e^x \sin x$

6.) $x^2 \sin x$ is an antiderivative since
$D(x^2 \sin x) = x^2 \cos x + 2x \sin x$

$\boxed{\text{SA5}}$: a) Rate $r = k \cdot p(A-p)$, where
$k$ is a proportionality constant (+).

b.) Maximize rate $r \rightarrow$

$r = kp(A-p) \xrightarrow{D} r' = kp(-1) + k(A-p) \rightarrow$

$r' = -kp + kA - kp = kA - 2kp = k(A-2p) = 0$

$r'$: $+$ | $0$ at $p = \frac{A}{2}$ | $-$

so $p = \frac{A}{2}$
determines a
maximum rate $r = \frac{kA^2}{4}$.

Math 16B
Kouba
Gravity Problems (The following problems ignore the effects of air friction and theoretical terminal velocities of falling objects.). Begin each problem with the acceleration due to gravity of $s''(t) = -32$ ft./sec$^2$ . Then solve for instantaneous velocity $s'(t)$ and height $s(t)$ .

1.) Assume that the vertical motion of an object has initial velocity, $v_0$, and initial height, $s_0$. Let $s(t)$ be the height (feet) of the object above the ground at time $t$ (seconds). Begin with the acceleration due to gravity of $-32$ ft/sec.$^2$ and derive the height equation $s(t) = -16t^2 + v_0 t + s_0$ .

2.) A baseball is projected *upward* from the top of a 128-ft. high building at 112 ft/sec.

a.) How high does the baseball go ?
b.) How long is the baseball in the air ?
c.) What is the baseball's velocity when $t = 3$ seconds ? $t = 4$ seconds? the ball strikes the ground ?

3.) A hard-boiled egg is projected *downward* from the top of a 320-ft. high building at 16 ft/sec.

a.) In how many seconds will the egg strike the ground ?
b.) What is the egg's velocity when $t = 1$ second ? $t = 2$ seconds? the egg strikes the ground ?

4.) An avocado is thrown *upward* from ground level and reaches its highest point in two seconds.

a.) How high does the avocado go ?
b.) What is the avocado's initial velocity ?

5.) A rock falls from a 1600-ft. high cliff.

a.) In how many seconds will the rock strike the ground ?
b.) What is the rock's velocity when $t = 5$ seconds ? the rock strikes the ground (in ft./sec. and miles/hr. where 1 mile = 5280 ft.) ?

6.) A watermelon is thrown *upward* from ground level and reaches a maximum height of 144 feet.

a.) How long does it take the watermelon to reach its highest point ?
b.) How long is the watermelon in the air ?

c.) What is the watermelon's initial velocity ?
d.) What will happen to the watermelon when it strikes the ground ?

7.) A bowling ball falls from an airplane at an elevation of 8000 feet.

a.) How long will it take the ball to reach an elevation of 1600 feet ?
b.) What is the ball's velocity at the elevation of 1600 feet ?

8.) A bottle of Snapple is thrown *downward* from a hovering helicopter from an unknown height and with an unknown initial velocity. The bottle falls from a height of 4000 feet to a height of 2400 feet in five (5) seconds and its velocity when $t = 10$ seconds is $-400$ ft./sec.

a.) What is the bottle's initial velocity ?
b.) What is the bottle's initial height ?
c.) How long is the bottle in the air ?
d.) What is your favorite kind of Snapple drink ?

9.) A water balloon is dropped from the top of a dormitory building. It strikes the ground in 5 seconds.

a.) How high is the building ?
b.) What is the balloon's velocity after 1 second ? after 3 seconds ?
c.) What is the balloon's velocity as it strikes the ground (in ft./sec) ? (in miles per hour) ?

# Gravity Problems

1.) acceleration : $s''(t) = -32 \rightarrow$

velocity : $s'(t) = -32t + c$

(Assume $s'(0) = v_0$.)

$\rightarrow v_0 = -32(0) + c \rightarrow c = v_0$

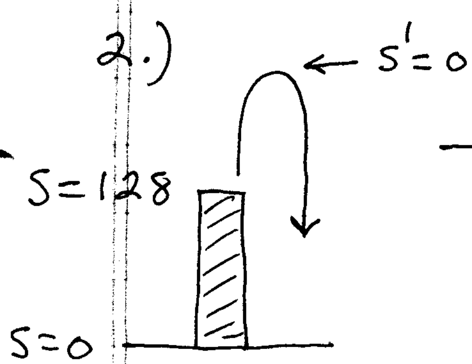
$\rightarrow s'(t) = -32t + v_0 \rightarrow$

height : $s(t) = -16t^2 + v_0 t + c$

(Assume $s(0) = s_0$.)

$\rightarrow s(t) = -16t^2 + v_0 t + s_0$.

2.)




assume $s(t) = -16t^2 + 112t + 128$

$\rightarrow$ vel. $\boxed{s'(t) = -32t + 112}$

a.) highest pt. : $s'(t) = 0 \rightarrow$

$-32t + 112 = 0 \rightarrow t = \frac{112}{32} = 3.5$ sec.

and $s(3.5) = -16(3.5)^2 + 112(3.5) + 128 = \boxed{324 \text{ ft.}}$

b.) strike ground : $s(t) = 0 \rightarrow$

$-16t^2 + 112t + 128 = -16(t^2 - 7t - 8)$

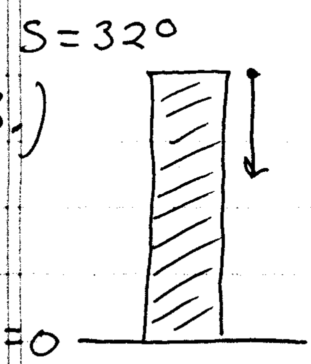
$= -16(t-8)(t+1) = 0 \rightarrow \boxed{t = 8 \text{ sec.}}$

c.) $s'(3) = 16$ ft./sec.,

$s'(4) = -16$ ft./sec.,

$s'(8) = -144$ ft./sec.

3.)




assume $s(t) = -16t^2 - 16t + 320$

$\rightarrow$ vel. $\boxed{s'(t) = -32t - 16}$

a.) strike ground : $s(t) = 0 \rightarrow$

$-16t^2 - 16t + 320 = -16(t^2 + t - 20)$

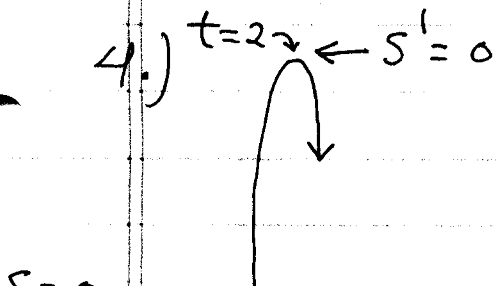
$= -16(t-4)(t+5) = 0 \rightarrow \boxed{t = 4 \text{ sec.}}$

b.) $s'(1) = -48$ ft./sec.,

$s'(2) = -80$ ft./sec.,

$s'(4) = -144$ ft./sec.

4.)




assume $s(t) = -16t^2 + v_0 t + (0)$

$\rightarrow s(t) = -16t^2 + v_0 t$

$\rightarrow$ vel. $\boxed{s'(t) = -32t + v_0}$

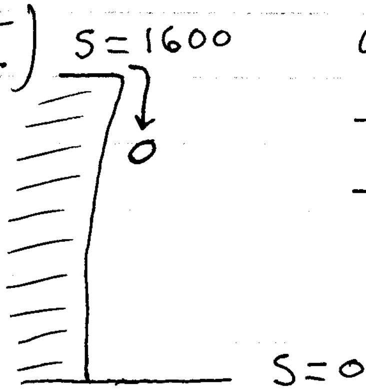
a.) $s'(2) = -32(2) + v_0 = 0 \rightarrow$

$v_0 = 64 \rightarrow \quad s(t) = -16t^2 + 64t \rightarrow$

$\boxed{s(2) = 64 \text{ ft.}}$

b.) $\boxed{v_0 = 64 \text{ ft./sec.}}$

5.)




assume $s(t) = -16t^2 + (0)t + 1600$

$\rightarrow s(t) = -16t^2 + 1600$

$\rightarrow$ vel. $s'(t) = -32t$

a.) strikes ground : $s(t) = 0 \rightarrow$

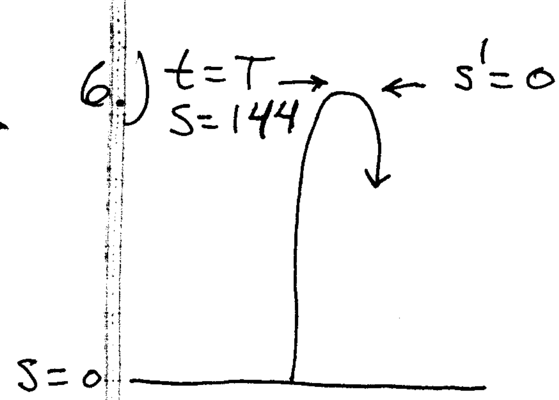
$-16t^2 + 1600 = -16(t^2 - 100)$
$= -16(t-10)(t+10) = 0 \rightarrow \boxed{t = 10 \text{ sec.}}$

b.) $s'(5) = -160$ ft./sec.,
$s'(10) = \boxed{-320 \text{ ft/sec.}}$

$= \dfrac{-320 \text{ ft.}}{\text{sec.}} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \dfrac{3600 \text{ sec.}}{1 \text{ hr.}}$

$\approx \boxed{-218.2 \text{ mph}}$

6.)




assume $s(t) = -16t^2 + V_0 t$
$\rightarrow$ vel. $s'(t) = -32t + V_0$

a.) $s'(T) = \boxed{-32T + V_0 = 0}$

and $S(T) = \boxed{-16T^2 + V_0 T = 144}$ ;

(Solve 2 equations, 2 unknowns)

$$\begin{cases} V_0 = 32T \\ -16T^2 + V_0 T = 144 \rightarrow -16T^2 + (32T)T = 144 \end{cases}$$

$\rightarrow 16T^2 = 144 \rightarrow T^2 = 9 \rightarrow T = 3$

so $\boxed{T = 3 \text{ sec.}}$ to reach highest point ;

and $V_0 = 32(3) \rightarrow \boxed{V_0 = 96 \text{ ft./sec.}}$

b.) strike ground : $S(t) = 0 \rightarrow$

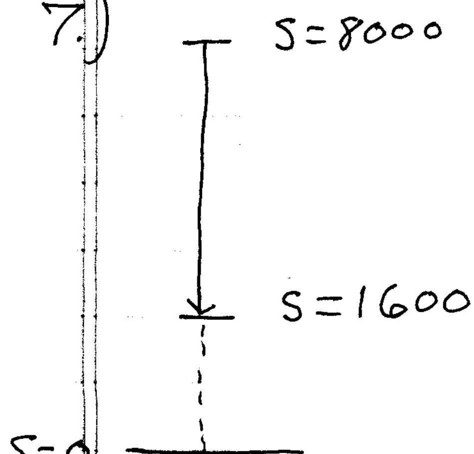
$s(t) = -16t^2 + 96t = 0 \rightarrow$

$-16t(t-6) = 0 \rightarrow \boxed{t = 6 \text{ sec.}}$

c.) $s'(0) = -32(0) + V_0 = \boxed{V_0 = 96 \text{ ft./sec.}}$

d.) You know.

7.)




assume $s(t) = -16t^2 + (0)t + 8000$

$\rightarrow s(t) = -16t^2 + 8000$

$\rightarrow$ vel. $s'(t) = -32t$

a.) $s(t) = 1600 \rightarrow$

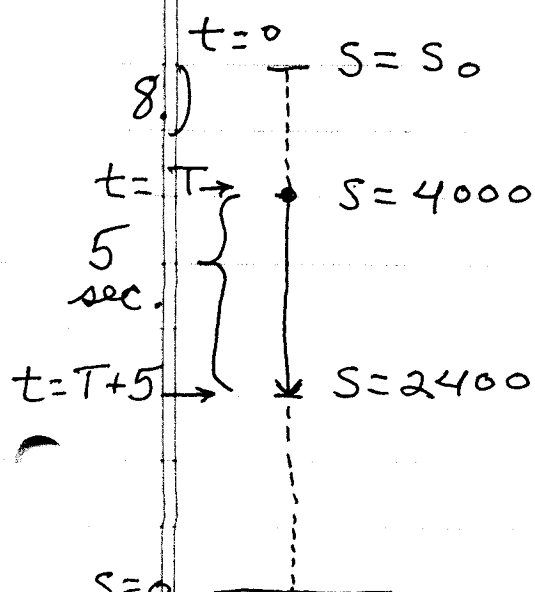
$-16t^2 + 8000 = 1600 \rightarrow$

$6400 = 16t^2 \rightarrow$

$t^2 = 400 \rightarrow \boxed{t = 20 \text{ sec.}}$

b.) $s'(20) = \boxed{-640 \text{ ft./sec.}}$

8.)




assume $s(t) = -16t^2 + V_0 t + S_0$

$\rightarrow$ vel. $s'(t) = -32t + V_0$ ;

a.) Given $s'(10) = -400 \rightarrow$

$-32(10) + V_0 = -400 \rightarrow$

$\boxed{V_0 = -80 \text{ ft./sec.}}$

$\rightarrow \boxed{s(t) = -16t^2 - 80t + S_0}$ ;

$S(T) = 4000 \rightarrow \begin{cases} -16T^2 - 80T + S_0 = 4000 \\ -16(T+5)^2 - 80(T+5) + S_0 = 2400 \end{cases}$
$S(T+5) = 2400 \rightarrow$

$\rightarrow \begin{cases} S_0 = 4000 + 16T^2 + 80T \quad \text{(SUB)} \\ -16(T^2 + 10T + 25) - 80T - 400 + S_0 = 2400 \end{cases}$

$\rightarrow -\cancel{16T^2} - 160T - 400 - \cancel{80T}$
$- 400 + (4000 + \cancel{16T^2} + \cancel{80T}) = 2400$

$\rightarrow -160T + 3200 = 2400$
$\rightarrow 800 = 160T \rightarrow T = 5$ sec. $\rightarrow$

b.) $S_0 = 4000 + 16(5)^2 + 80(5) \rightarrow$

$\boxed{S_0 = 4800 \text{ ft.}}$ ; then

$\boxed{s(t) = -16t^2 - 80t + 4800}$

c.) strike ground : $s(t) = 0 \rightarrow$
$-16t^2 - 80t + 4800 = -16(t^2 + 5t - 300)$
$= -16(t-15)(t+20) = 0 \rightarrow \boxed{t = 15 \text{ sec}}$ ;

d.) peach tea, raspberry tea

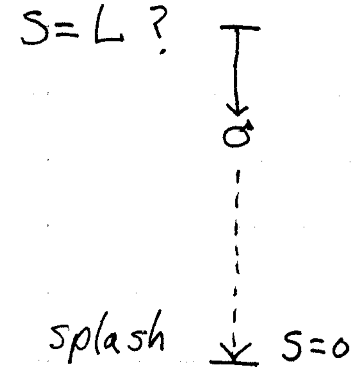
9.) $s''(t) = -32 \rightarrow$

$s'(t) = -32t + C$

$(t=0, s'=0 \rightarrow 0 = -32(0) + C$

$\rightarrow C = 0) \rightarrow$

$\boxed{s'(t) = -32t} \rightarrow$




$s(t) = -16t^2 + C \quad (t=0, s=L \rightarrow$

$L = -16(0)^2 + C \rightarrow C = L) \rightarrow$

$\boxed{s(t) = -16t^2 + L}$ ;

Given $s(5) = 0 \rightarrow -16(5)^2 + L = 0 \rightarrow$

$L = 400 \rightarrow \boxed{s(t) = -16t^2 + 400}$

a.) $s(0) = -16(0)^2 + 400 = 400$ ft.

b.) $s'(1) = -32(1) = -32$ ft./sec. ,

$s'(3) = -32(3) = -96$ ft./sec. ,

c.) $s'(5) = -32(5) = -160$ ft./sec. ;

$$\frac{-160 \text{ ft.}}{\text{sec.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx -109.1 \text{ mph}$$