Section 5.2

9.) $\int (1+2x)^4(2)\,dx$ (Let $u=1+2x \rightarrow du = 2\,dx$)

$= \int u^4\,du = \frac{1}{5}u^5 + c = \frac{1}{5}(1+2x)^5 + c$

12.) $\int \sqrt{3-x^3}\cdot(3x^2)\,dx$ (Let $u = 3-x^3 \rightarrow du = -3x^2\,dx$

$= \int \sqrt{u}\cdot{-du}$ $\rightarrow -du = 3x^2\,dx$)

$= -\int u^{1/2}\,du = -\frac{u^{3/2}}{3/2} + c = -\frac{2}{3}(3-x^3)^{3/2} + c$

15.) $\int x(x^2-1)^7\,dx$ (Let $u = x^2-1 \rightarrow du = 2x\,dx$

$= \int u^7\cdot\frac{1}{2}\,du$ $\rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\cdot\frac{1}{8}u^8 + c = \frac{1}{16}(x^2-1)^8 + c$

18.) $\int \frac{x^2}{(x^3-1)^2}\,dx$ (Let $u = x^3-1 \rightarrow du = 3x^2\,dx$

$= \int \frac{\frac{1}{3}}{u^2}\,du$ $\rightarrow \frac{1}{3}du = x^2\,dx$)

$= \frac{1}{3}\int u^{-2}\,du = \frac{1}{3}\cdot\frac{u^{-1}}{-1} + c = \frac{-1}{3}(x^3-1)^{-1} + c$

21.) $\int \frac{x-2}{\sqrt{x^2-4x+3}}\,dx$ (Let $u = x^2-4x+3 \rightarrow$

$du = (2x-4)\,dx = 2(x-2)\,dx \rightarrow$

$= \int \frac{\frac{1}{2}}{\sqrt{u}}\,du$ $\frac{1}{2}du = (x-2)\,dx$)

$= \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\cdot\frac{u^{1/2}}{\frac{1}{2}} + c = (x^2-4x+3)^{1/2} + c$

24.) $\int u^3\sqrt{u^4+2}\,du$ (Let $x = u^4+2 \rightarrow dx = 4u^3\,du$

$= \int \sqrt{x}\cdot\frac{1}{4}\,dx$ $\rightarrow \frac{1}{4}dx = u^3\,du$)

$= \frac{1}{4}\cdot\frac{x^{3/2}}{3/2} + c = \frac{1}{6}\cdot(u^4+2)^{3/2} + c$

27.) $\int \frac{-3}{\sqrt{2x+3}}\,dx$ (Let $u = 2x+3 \rightarrow du = 2\,dx \rightarrow$

$\frac{1}{2}du = dx$)

$$= \int \frac{-3}{\sqrt{u}} \cdot \frac{1}{2}\, du = \frac{-3}{2} \int u^{-1/2} du = \frac{-3}{2} \cdot \frac{u^{1/2}}{1/2} + c$$

$$= -3(2x+3)^{1/2} + c$$

32.) $\int \left(1+\frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$ (Let $u = 1 + \frac{1}{t} \rightarrow du = \frac{-1}{t^2}\, dt \rightarrow -du = \frac{1}{t^2}\, dt$)

$$= \int u^3 \cdot -du$$

$$= -\frac{1}{4} u^4 + c = \frac{-1}{4}\left(1+\frac{1}{t}\right)^4 + c$$

33.) $\int (x^3 + 3x)(x^2+1)\, dx$ (Let $u = x^3 + 3x \rightarrow du = (3x^2+3)\, dx = 3(x^2+1)\, dx \rightarrow \frac{1}{3} du = (x^2+1)\, dx$

$$= \int u \cdot \frac{1}{3} du$$

$$= \frac{1}{3} \int u\, du$$

$$= \frac{1}{3} \cdot \frac{u^2}{2} + c = \frac{1}{6}(x^3 + 3x)^2 + c$$

41.) $\int \frac{x^2+1}{\sqrt{x^3+3x+4}}\, dx$ (Let $u = x^3 + 3x + 4 \rightarrow du = (3x^2+3)\, dx = 3(x^2+1)\, dx \rightarrow \frac{1}{3} du = (x^2+1)\, dx$)

$$= \int \frac{\frac{1}{3}}{\sqrt{u}}\, du$$

$$= \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} + c = \frac{2}{3}(x^3 + 3x + 4)^{1/2} + c$$

42.) $\int \sqrt{x}\,(4 - x^{3/2})^2\, dx$ (Let $u = 4 - x^{3/2} \rightarrow du = -\frac{3}{2} x^{1/2} dx \rightarrow -\frac{2}{3} du = \sqrt{x}\, dx$)

$$= \int u^2 \cdot \frac{-2}{3} du$$

$$= \frac{-2}{3} \cdot \frac{u^3}{3} + c = \frac{-2}{9}(4 - x^{3/2})^3 + c$$

47.) $f'(x) = x\sqrt{1-x^2} \rightarrow f(x) = \int x\sqrt{1-x^2}\, dx \rightarrow$

(Let $u = 1 - x^2 \rightarrow du = -2x\, dx \rightarrow \frac{-1}{2} du = x\, dx$)

$f(x) = \int \sqrt{u} \cdot \frac{-1}{2} du = \frac{-1}{2} \cdot \frac{u^{3/2}}{3/2} + c = -\frac{1}{3}(1-x^2)^{3/2} + c$ and

$f(0) = \frac{4}{3} \rightarrow \frac{4}{3} = \frac{-1}{3}(1)^{3/2} + c \rightarrow c = 5/3 \rightarrow$

$$f(x) = \frac{-1}{3}(1-x^2)^{3/2} + 5/3 .$$

55.) $\frac{dh}{dt} = \frac{17.6t}{\sqrt{17.6t^2+1}} \rightarrow h = \int \frac{17.6t}{\sqrt{17.6t^2+1}}\,dt$

$= \int \frac{17.6}{\sqrt{u}} \cdot \frac{1}{35.2}\,du$ (Let $u = 17.6t^2+1 \rightarrow$

$du = 35.2t\,dt \rightarrow \frac{1}{35.2}\,du = t\,dt$)

$= \frac{1}{2}\int u^{-1/2}\,du$

$= \frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + c = \sqrt{17.6t^2+1} + c \rightarrow$

$h = \sqrt{17.6t^2+1} + c$ and $t=0, h=6 \rightarrow$

$6 = \sqrt{17.6(0)^2+1} + c \rightarrow 6 = 1 + c \rightarrow c = 5 \rightarrow$

a.) $h = \sqrt{17.6t^2+1} + 5$

b.) $t = 5 \rightarrow h = \sqrt{17.6(5)^2+1} + 5 = 26$ in.

59.) $\int \frac{1}{\sqrt{x}+\sqrt{x+1}}\,dx = \int \frac{1}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}-\sqrt{x+1}}\,dx$

$= \int \frac{\sqrt{x}-\sqrt{x+1}}{x-(x+1)}\,dx = \int (\sqrt{x+1}-\sqrt{x})\,dx = \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + c$

60.) $\int \frac{x}{\sqrt{3x+2}}\,dx$ (Let $u = 3x+2 \rightarrow du = 3\,dx \rightarrow$

$\frac{1}{3}\,du = dx$ and $x = \frac{1}{3}(u-2)$)

$= \int \frac{\frac{1}{3}(u-2)}{\sqrt{u}} \cdot \frac{1}{3}\,du$

$= \frac{1}{9}\int \frac{u-2}{u^{1/2}}\,du = \frac{1}{9}\int (u^{1/2} - 2u^{-1/2})\,du$

$= \frac{1}{9}\left(\frac{u^{3/2}}{3/2} - 2 \cdot \frac{u^{1/2}}{1/2}\right) + c = \frac{1}{9}\left(\frac{2}{3}(3x+2)^{3/2} - 4(3x+2)^{1/2}\right) + c$

$\boxed{SA6}$: a.) $Y = 7e^{X^3+1} - 2$ (switch x, Y) $\rightarrow$

$x = 7e^{Y^3+1} - 2$ (solve for Y) $\rightarrow$

$x+2 = 7e^{Y^3+1} \rightarrow \ln\left(\frac{x+2}{7}\right) = \ln e^{Y^3+1} \rightarrow$

$\ln\left(\frac{x+2}{7}\right) = Y^3+1 \rightarrow Y^3 = \ln\left(\frac{x+2}{7}\right) - 1 \rightarrow$

$Y = \left[\ln\left(\frac{x+2}{7}\right) - 1\right]^{1/3}$ is inverse function.

b.) $Y = 3\log_7(4X-5)$ (switch x, Y) $\rightarrow$

$x = 3\log_7(4Y-5)$ (solve for Y) $\rightarrow$

$\frac{x}{3} = \log_7(4Y-5) \rightarrow 4Y-5 = 7^{x/3} \rightarrow$

$4Y = 5 + 7^{x/3} \rightarrow Y = \frac{1}{4}\left(5 + 7^{x/3}\right)$ is inverse function.

c.) $Y = \ln(x+1) - \ln(x+2)$ (switch x, y) $\rightarrow$

$x = \ln(Y+1) - \ln(Y+2)$ (solve for Y) $\rightarrow$

$x = \ln\frac{Y+1}{Y+2} \rightarrow \frac{Y+1}{Y+2} = e^x \rightarrow Y+1 = Ye^x + 2e^x \rightarrow$

$Y - Ye^x = 2e^x - 1 \rightarrow Y(1-e^x) = 2e^x - 1 \rightarrow$

$Y = \frac{2e^x - 1}{1 - e^x}$ is inverse function.