Math 16B
Kouba
Estimating the Value of a Definite Integral

Suppose that the integral $\int_a^b f(x)\,dx$ is too difficult (or impossible) to compute, or that you are simply required to estimate its exact value. The following two methods offer two different ways to determine good estimates.

1.) MIDPOINT RULE

a.) Divide the interval $[a, b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval and let the sampling points $c_1, c_2, c_3, ..., c_n$ be the MIDPOINTS of these subintervals.

c.) The Midpoint Estimate for $\int_a^b f(x)\,dx$ is

$$M_n = h\,[f(c_1) + f(c_2) + f(c_3) + ... + f(c_n)]\ .$$

2.) TRAPEZOIDAL RULE

a.) Divide the interval $[a, b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval.

c.) The Trapezoidal Estimate for $\int_a^b f(x)\,dx$ is

$$T_n = \frac{h}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)]\ .$$