Math 16B
Kouba
Estimating the Value of a Definite Integral

Suppose that the integral $\int_a^b f(x)\,dx$ is too difficult (or impossible) to compute, or that you are simply required to estimate its exact value. The following three methods offer three different ways to compute an estimate.

1.) MIDPOINT RULE

a.) Divide the interval $[a,b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval and let the sampling points $c_1, c_2, c_3, ..., c_n$ be the MIDPOINTS of these subintervals.

c.) The Midpoint Estimate for $\int_a^b f(x)\,dx$ is

$$M_n = h\ [f(c_1) + f(c_2) + f(c_3) + ... + f(c_n)]\ .$$

d.) The Absolute Error is $|E_n| \le (b-a)h\left\{ \max_{a \le x \le b} |f'(x)| \right\}$.

2.) TRAPEZOIDAL RULE

a.) Divide the interval $[a,b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval.

c.) The Trapezoidal Estimate for $\int_a^b f(x)\,dx$ is

$$T_n = \frac{h}{2}\ [f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)]\ .$$

d.) The Absolute Error is $|E_n| \le (b-a)\dfrac{h^2}{12}\left\{ \max_{a \le x \le b} |f''(x)| \right\}$.

3.) SIMPSON'S RULE (NOTE: For this method n MUST be an even integer !)

a.) Divide the interval $[a,b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval.

c.) The Simpson Estimate for $\int_a^b f(x)\,dx$ is

$$S_n = \frac{h}{3}\ [f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+...+4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$$

d.) The Absolute Error is $|E_n| \le (b-a)\dfrac{h^4}{180}\left\{ \max_{a \le x \le b} |f^{(4)}(x)| \right\}$.