Math 16B
Section 4.3

Differentiating Exponentials

Rules: 1.) $D(e^x) = e^x$

and 2.) $D(e^{f(x)}) = e^{f(x)} \cdot f'(x)$

(These will be proven later.)

Example: Differentiate each function.

1.) $f(x) = x^2 e^x \xrightarrow{D}$

$f'(x) = x^2 e^x + (2x) e^x$

2.) $y = 3e^{x^2} \xrightarrow{D} y' = 3 \cdot e^{x^2} \cdot 2x$

3.) $y = \dfrac{5x}{3+e^{-x}} \xrightarrow{D}$

$$y' = \frac{(3+e^{-x})(5) - (5x)(-e^{-x})}{(3+e^{-x})^2}$$

4.) $f(x) = (x - e^{3x})^{-4} \xrightarrow{D}$

$f'(x) = -4(x - e^{3x})^{-5} \cdot (1 - 3e^{3x})$

5.) $y = \tan(e^{10x}) \xrightarrow{D}$

$y' = \sec^2(e^{10x}) \cdot e^{10x} \cdot 10$

6.) $y = \sin(e^{\sec(x^2)}) \xrightarrow{D}$

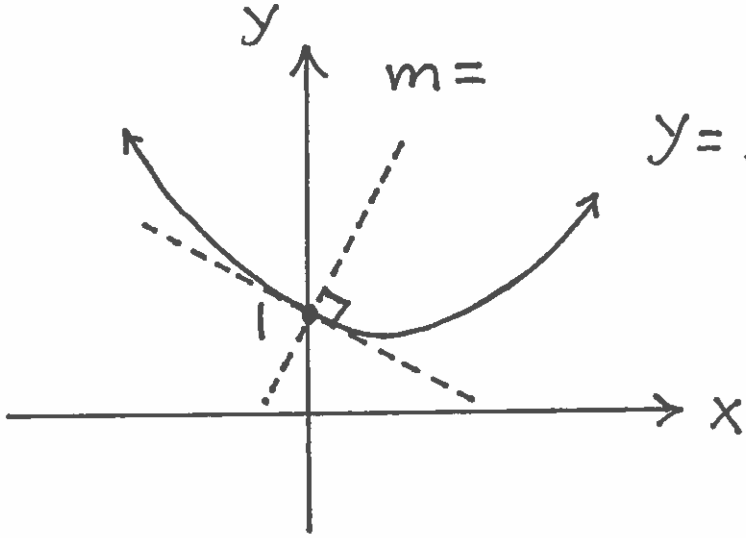
$y' = \cos(e^{\sec(x^2)}) \cdot e^{\sec(x^2)} \cdot$

$\sec(x^2)\tan(x^2) \cdot 2x$

7.) $f(x) = e^{2x} \cdot (x - e^{x^2}) \xrightarrow{D}$

$f'(x) = e^{2x}(1 - e^{x^2} \cdot 2x) + 2e^{2x}(x - e^{x^2})$

<u>Example</u>: Find an equation of the line perpendicular to the graph of $y = 3e^{-x} + x$ at $x = 0$.



$y = 3e^{-x} + x$,

$x = 0$:

$y = 3 \cdot e^{0} + 0 = 3$,

$\xrightarrow{D}$ $y' = -3e^{-x} + 1$, and $x = 0$ so SLOPE of tangent line is $y' = -3e^{0} + 1 = -2$; then SLOPE of $(\perp)$ line is $m = \frac{1}{2}$, so $(\perp)$ line is

$y - 3 = \frac{1}{2}(x - 0)$ or $y = \frac{1}{2}x + 3$

<u>Example</u>: Solve $y' = 0$ for $y = x^2 e^{2x}$.

$\xrightarrow{D}$ $y' = x^2 \cdot 2e^{2x} + 2xe^{2x}$

$= 2xe^{2x}(x + 1) = 0 \longrightarrow$

$x = 0, \ x = -1$

__Example__: Do Detailed Graphing for $f(x) = xe^x$: Domain : all x-values

$$\xrightarrow{D} f'(x) = xe^x + (1)e^x = e^x(x+1) = 0$$

$$\underset{\rule{8em}{0.4pt}}{-\quad 0 \quad +} \quad f'$$

ABS. MIN. $\begin{cases} x = -1 \\ y = \frac{-1}{e} \end{cases}$

$$\xrightarrow{D} y'' = e^x(1) + e^x(x+1)$$
$$= e^x(1+x+1) = e^x(2+x) = 0$$

$$\underset{\rule{8em}{0.4pt}}{-\quad 0 \quad +} \quad f''$$

Infl. pt. $\begin{cases} x = -2 \\ y = \frac{-2}{e^2} \end{cases}$

$x = 0$: $y = 0$

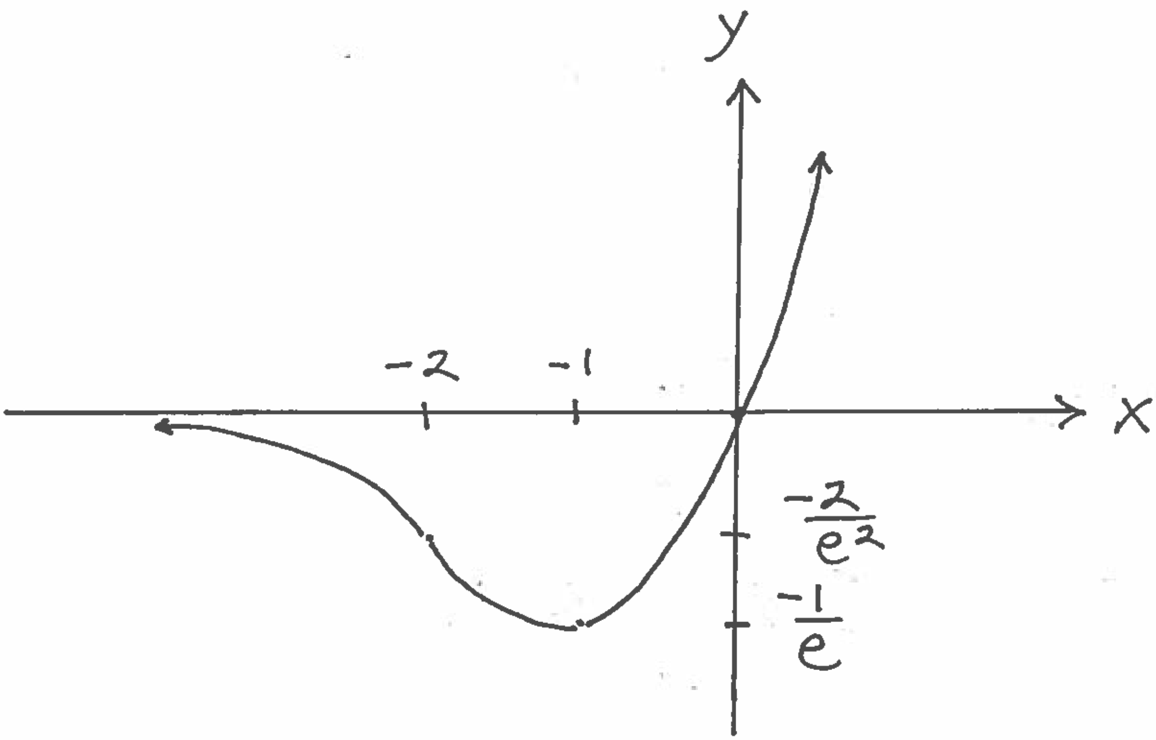
$y = 0$: $xe^x = 0 \rightarrow x = 0$

y is $(\uparrow)$ for $x > -1$,

y is $(\downarrow)$ for $x < -1$,

y is $(\cup)$ for $x > -2$,

y is $(\cap)$ for $x < -2$



<u>Example</u>: assume that y is a function of x and $e^y + xy^2 = e^{2x}$. Find $y' = \frac{dy}{dx}$.

$$\xrightarrow{D} \quad e^y \cdot y' + (x \cdot 2yy' + (1)y^2) = 2e^{2x}$$

$$\longrightarrow \quad (e^y + 2xy)\, y' = 2e^{2x} - y^2$$

$$\longrightarrow \quad y' = \frac{2e^{2x} - y^2}{e^y + 2xy}$$