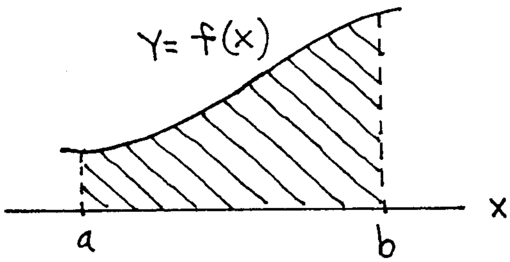
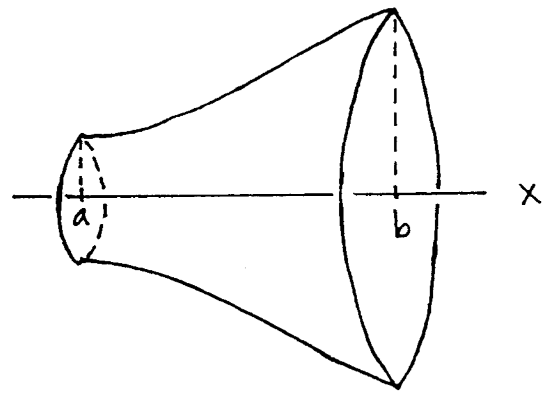
Math 16B
Kouba
Volumes of Solids of Revolutions

Consider the shaded region bounded by the graph of $y = f(x)$ and the x-axis on the interval [a, b]. Create a three-dimensional "cylinder" by revolving this region about the x-axis. We will derive a formula for the volume of the resulting solid.



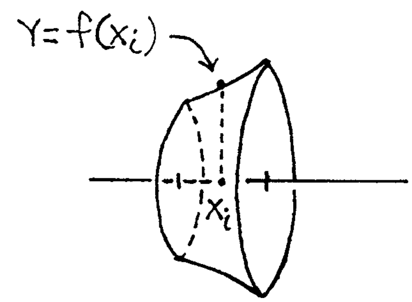


1.) Divide the interval [a, b] into n equal parts each of length $\frac{b-a}{n}$.

2.) Label the midpoints of the n subintervals : $x_1, x_2, x_3, \ldots, x_n$.

3.) An *estimate* for the volume of the i-th slice, which is approximately a cylinder ($V = \pi r^2 h$),

is $\pi\,[\,f(x_i)\,]^2 \cdot \frac{b-a}{n}$



4.) By adding up the volume estimates for all n slices, then taking the limit as n goes to infinity, we get a formula for the exact volume of this solid :

$$\text{Volume} = \lim_{n \to \infty} \frac{b-a}{n} \left\{ \pi\,[f(x_1)]^2 + \pi\,[f(x_2)]^2 + \pi\,[f(x_3)]^2 + \cdots + \pi\,[f(x_n)]^2 \right\}$$

$$= \int_a^b \pi\,[\,f(x)\,]^2\,dx$$

$$= \pi \int_a^b [\,f(x)\,]^2\,dx$$