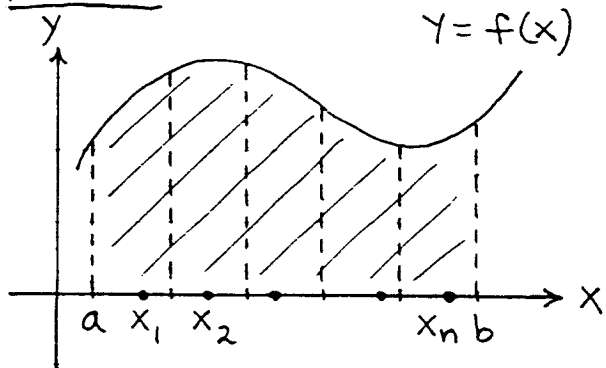


Math 16C

Kouba

Double Integrals

Recall:

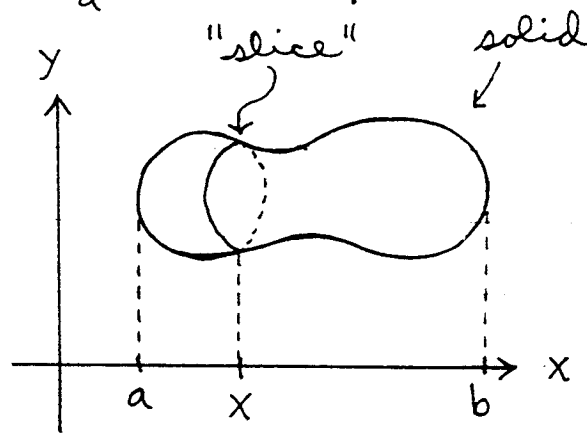


Divide interval $[a, b]$ into n equal parts each of length $\frac{b-a}{n}$. Then the area of the shaded region is given by

$$\text{Area} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \frac{b-a}{n} = \int_a^b f(x) dx$$

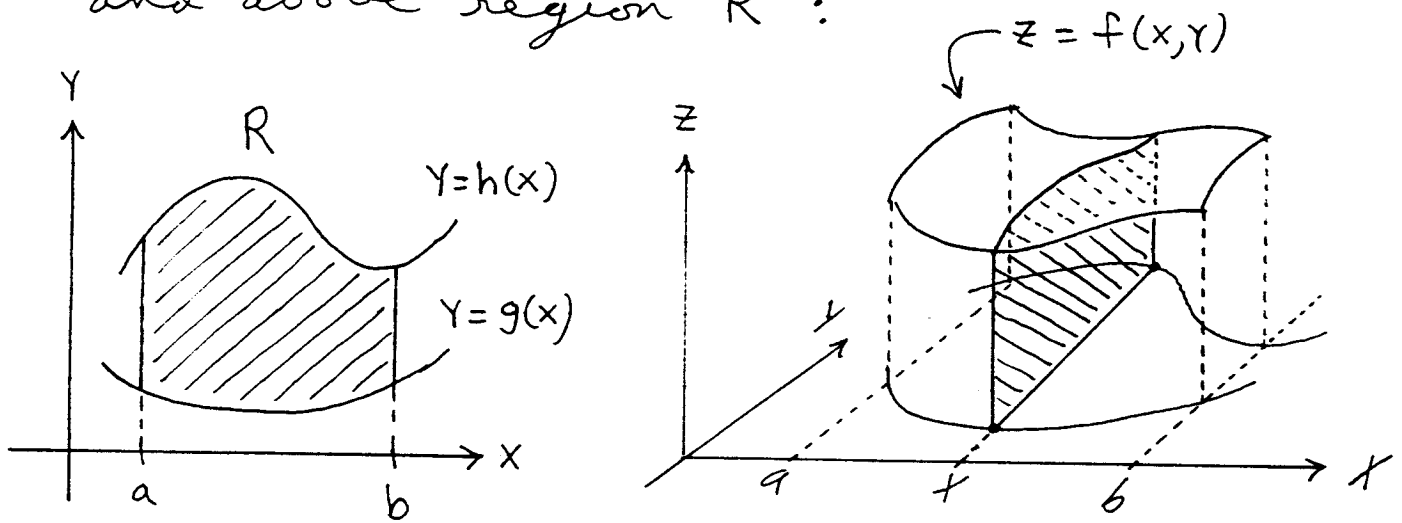
Fact: If $A(x)$ is the area of a "slice" of a solid taken perpendicular to the x -axis at x , then the volume of the solid is given by

$$\text{Volume} = \int_a^b A(x) dx$$



Double Integral: Consider region R bounded by $a \leq x \leq b$ and $g(x) \leq y \leq h(x)$ and let $z = f(x, y)$ be a surface defined on R . We seek to compute the volume of the

solid region lying below the surface and above region R :



Pick an x -value and make a slice perpendicular to the x -axis. Let $A(x)$ be the area of this slice. Then

$$A(x) = \int_{g(x)}^{h(x)} f(x,y) dy$$

and the volume of the solid is

$$\text{Volume} = \int_a^b A(x) dx, \text{ or}$$

$$\text{Volume} = \int_a^b \int_{g(x)}^{h(x)} f(x,y) dy dx$$