Math 16C
Kouba
Geometric Series

Fact 1: $1+r+r^2+r^3+\cdots+r^n = \dfrac{1-r^{n+1}}{1-r}$ for any constant $r \neq 1$.

proof: Let $S = 1+r+r^2+r^3+\cdots+r^n$.
Then $rS = r+r^2+r^3+\cdots+r^n+r^{n+1}$.

Subtract: $S - rS = 1-r^{n+1} \Rightarrow (1-r)S = 1-r^{n+1} \Rightarrow$

$$S = \frac{1-r^{n+1}}{1-r}.$$

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Fact 2: $\lim_{n\to\infty} (r)^n = 0$ if $-1<r<1$; $\lim_{n\to\infty} (r)^n = \infty$ if $r>1$; $\lim_{n\to\infty} (r)^n$ does not exist if $r \leq -1$.

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Fact 3: $1+r+r^2+r^3+\cdots = \dfrac{1}{1-r}$ for $-1<r<1$.
The infinite series diverges for all other values of r.

proof: $1+r+r^2+r^3+\cdots = \lim_{n\to\infty}(1+r+r^2+\cdots+r^n)$

$$= \lim_{n\to\infty} \frac{1-r^{n+1}}{1-r} = \frac{1-0}{1-r} = \frac{1}{1-r} \text{ for } -1<r<1.$$

This limit does not exist for all other values of r.

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