Section 7.5

33.) Let x, y, z be positive #'s :

$x + y + z = 30 \rightarrow z = 30 - x - y$;

maximize product

$P = xyz = xy(30 - x - y) \rightarrow$

$\boxed{P = 30xy - x^2y - xy^2}$; then

$P_x = 30y - 2xy - y^2$

$= y(30 - 2x - y) = 0 \rightarrow$

(No) $\boxed{\cancel{y = 0}}$ or $30 - 2x - y = 0 \rightarrow \boxed{y = 30 - 2x}$;

and $P_y = 30x - x^2 - 2xy$

$= x(30 - x - 2y) = 0 \rightarrow$

(No) $\boxed{\cancel{x = 0}}$ or $\boxed{30 - x - 2y = 0}$; then

(sub.) $30 - x - 2(30 - 2x) = 0 \rightarrow$

$30 - x - 60 + 4x = 0 \rightarrow 3x - 30 = 0 \rightarrow$

$x = 10$, $y = 10$, $z = 10$ and $P = 1000$

34.) Let x, y, z be positive #'s :

$x + y + z = 32 \rightarrow z = 32 - x - y$;

maximize $P = xy^2z = xy^2(32 - x - y)$

$\rightarrow \boxed{P = 32xy^2 - x^2y^2 - xy^3}$; then

$P_x = 32y^2 - 2xy^2 - y^3$

$= y^2(32 - 2x - y) = 0 \rightarrow \boxed{\cancel{y = 0}}$ (No)

or $32 - 2x - y = 0 \rightarrow \boxed{y = 32 - 2x}$; and

$P_y = 64xy - 2x^2y - 3xy^2$

$= xy(64 - 2x - 3y) = 0 \rightarrow \boxed{\cancel{x = 0}}$ (No)

or $\boxed{\cancel{y = 0}}$ (No) or $\boxed{64 - 2x - 3y = 0}$; ~~then~~

(sub.) $64-2X-3(32-2X)=0 \rightarrow$

$64-2X-96+6X=0 \rightarrow 4X-32=0 \rightarrow$

$X=8$, $Y=16$, and $Z=8$ and $P=16,384$.

35.) Let X, Y, Z be positive #'s :

$X+Y+Z=30 \rightarrow Z=30-X-Y$;

minimize $S=X^2+Y^2+Z^2 \rightarrow$

$\boxed{S=X^2+Y^2+(30-X-Y)^2}$; then

$$S_X = 2X+2(30-X-Y)(-1)$$
$$= 2X-60+2X+2Y$$
$$= 4X+2Y-60 = 2(2X+Y-30)=0 \rightarrow$$

$2X+Y-30=0 \rightarrow \boxed{Y=30-2X}$; and

$$S_Y = 2Y+2(30-X-Y)(-1)$$
$$= 2Y-60+2X+2Y$$
$$= 4Y+2X-60 = 2(2Y+X-30)=0 \rightarrow$$

$\boxed{2Y+X-30=0}$; then (sub.)

$2(30-2X)+X-30=0 \rightarrow 60-4X+X-30=0$

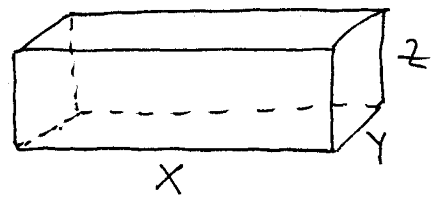
$\rightarrow 30=3X \rightarrow$ $X=10$, $Y=10$, $Z=10$.

37.) $R=-5x_1^2-8x_2^2-2x_1x_2+42x_1+102x_2 \rightarrow$

$$\left.\begin{aligned} R_{x_1} &= -10x_1-2x_2+42=0 \\ R_{x_2} &= -16x_2-2x_1+102=0 \end{aligned}\right\} \quad \left.\begin{aligned} x_2 &= 21-5x_1 \\ x_1 &= 51-8x_2 \end{aligned}\right\}$$

$x_2=21-5(51-8x_2) \rightarrow x_2=21-255+40x_2 \rightarrow$

$234=39x_2 \rightarrow$ $x_2=6$ and $x_1=3$ and $R=\$369$

43.) 

$X + 2Y + 2Z = 144 \rightarrow$

$\underline{X = 144 - 2Y - 2Z}$; maximize volume

$V = XYZ = (144 - 2Y - 2Z)YZ \rightarrow$

$\boxed{V = 144YZ - 2Y^2Z - 2YZ^2}$; then

$V_y = 144Z - 4YZ - 2Z^2 = 2Z(72 - 2Y - Z) = 0$

$\rightarrow Z = 0$ (NO) or $72 - 2Y - Z = 0 \rightarrow \boxed{Z = 72 - 2Y}$;

$V_Z = 144Y - 2Y^2 - 4YZ = 2Y(72 - Y - 2Z) = 0$

$\rightarrow Y = 0$ (NO) or $\boxed{72 - Y - 2Z = 0}$; then (sub)

$72 - Y - 2(72 - 2Y) = 0 \longrightarrow$

$72 - Y - 144 + 4Y = 0 \rightarrow 3Y = 72 \rightarrow$ $Y = 24$ in.,

$Z = 24$ in., $X = 48$ in. and volume

$V = 27{,}648$ in.^{3}.

46.) Total weight $T = T_{small} + T_{large}$

$= X(3 - 0.002X - 0.005Y)$

$+ Y(4.5 - 0.003X - 0.004Y) \rightarrow$

$T = 3X - 0.002X^2 - 0.005XY$

$+ 4.5Y - 0.003XY - 0.004Y^2 \rightarrow$

$\boxed{T = 3X + 4.5Y - 0.002X^2 - 0.004Y^2 - 0.008XY}$;

$T_x = 3 - 0.004X - 0.008y = 0 \rightarrow$

$3 - 0.004X = 0.008y \rightarrow \boxed{y = 375 - \frac{1}{2}X}$;

$T_y = 4.5 - 0.008Y - 0.008X = 0 \rightarrow$

$\boxed{4.5 - 0.008Y - 0.008X = 0}$; then (sub.)

$4.5 - 0.008(375 - \frac{1}{2}X) - 0.008X = 0 \rightarrow$

$4.5 - 3 - 0.004X - 0.008X = 0 \rightarrow$

$1.5 - 0.012X = 0 \rightarrow 1.5 = 0.012X \rightarrow$

$X = 125$ small, $y \approx 312$ large, and

$T \approx 1046.37$ lbs. (?)

Math 16C
Kouba
Worksheet 5

On each of the following problems you need only determine the desired critical point and its corresponding extreme value. Do not apply the second derivative test.

1.) Construct a closed rectangular box having a volume of 8 cubic feet. What are the dimensions of the box having a minimum surface area ? What is the minimum surface area ?

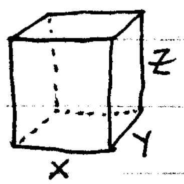
2.) Construct an open (no top) rectangular box from material which costs 3/4 cents per square foot for the bottom and 3 cents per square foot for the sides. What are the dimensions of the least expensive box having a volume of 1 cubic foot ? What is the minimum cost ?

3.) Determine the shortest distance from the origin to the plane $x + 2y + 3z = 6$.

4.) Determine the shortest distance between the graphs of $y = e^x$ and $y = x$. (This problem is somewhat challenging. It can be solved a couple of different ways.)

Worksheet 5

Volume 2

1.) 

$8 = XYZ \to \quad Z = \frac{8}{XY}$;

minimize surface area

$S = 2XY + 2YZ + 2XZ \to$

$$S = 2XY + 2Y\left(\frac{8}{XY}\right) + 2X\left(\frac{8}{XY}\right) = 2XY + \frac{16}{X} + \frac{16}{Y} \to$$

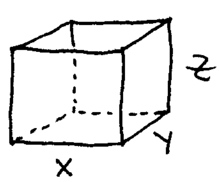
$$S_X = 2Y - \frac{16}{X^2} = 0 \to \quad Y = 8/X^2 \quad \text{and}$$

$$S_Y = 2X - \frac{16}{Y^2} = 0 \to \quad X = 8/Y^2 \quad \to$$

$$Y = \frac{8}{\left(\frac{8}{Y^2}\right)^2} = \frac{1}{8}Y^4 \to 8Y - Y^4 = Y(8 - Y^3) = 0 \to$$

$\cancel{Y=0}$ or $\boxed{Y=2}$, $\boxed{X=2}$, and $\boxed{Z=2}$ and

$S = 24\ ft^2$ is minimum surface area.

2.) 

Volume $\to 1 = XYZ \to Z = \frac{1}{XY}$;

minimize cost

$$C = \frac{3}{4}(XY) + 3(2XZ + 2YZ)$$
$$= \frac{3}{4}(XY) + 6XZ + 6YZ$$
$$= \frac{3}{4}XY + 6X\left(\frac{1}{XY}\right) + 6Y\left(\frac{1}{XY}\right)$$
$$= \frac{3}{4}XY + \frac{6}{Y} + \frac{6}{X} \quad \text{then}$$

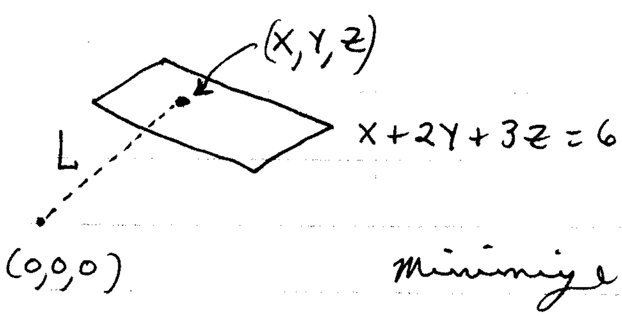
$$\left.\begin{aligned} C_X &= \frac{3}{4}Y - \frac{6}{X^2} = 0 \\ C_Y &= \frac{3}{4}X - \frac{6}{Y^2} = 0 \end{aligned}\right\} \quad \begin{aligned} Y &= \frac{8}{X^2} \\ X &= \frac{8}{Y^2} \end{aligned} \rightarrow X = \frac{8}{\left(\frac{8}{X^2}\right)^2} \rightarrow$$

$$X = \frac{1}{8}X^4 \rightarrow X^4 - 8X = 0 \rightarrow X(X^3 - 8) = 0 \rightarrow$$

$\cancel{X = 0}$ or $X = 2$ ft., $Y = 2$ ft., $Z = \frac{1}{4}$ ft. and

the minimum cost is $C = 9¢$.

3.)



minimize distance

$$L = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$
$$= \sqrt{(6 - 2y - 3z)^2 + y^2 + z^2} \longrightarrow$$

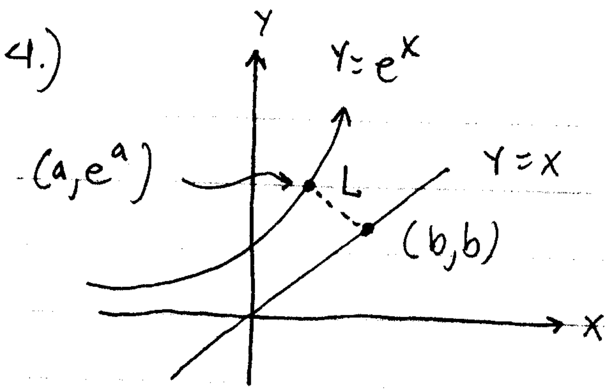
$$\left.\begin{aligned} L_y &= \tfrac{1}{2}(\sim\!\sim)^{-1/2} \cdot \left[2(6 - 2y - 3z)(-2) + 2y\right] = 0 \\ L_z &= \tfrac{1}{2}(\sim\!\sim)^{-1/2} \left[2(6 - 2y - 3z)(-3) + 2z\right] = 0 \end{aligned}\right\} \rightarrow$$

$$(6-2Y-3Z)(-2)+Y=0 \\ (6-2Y-3Z)(-3)+Z=0 \Big\} \quad -12+5Y+6Z=0 \\ -18+6Y+10Z=0 \Big\}$$

$$Y=\frac{1}{5}(12-6Z) \\ Y=\frac{1}{6}(18-10Z) \Big\} \quad \frac{1}{5}(12-6Z)=\frac{1}{6}(18-10Z) \rightarrow$$

$72-36Z=90-50Z \rightarrow 14Z=18 \rightarrow Z=9/7$, $Y=\frac{6}{7}$, and $X=3/7$ determine a minimum distance of $\boxed{L=1.60}$.

4.)



Minimize distance L given by

$$L=\sqrt{(a-b)^2+(e^a-b)^2} \rightarrow$$

$$L_a=\frac{1}{2}(\sim)^{-\frac{1}{2}}\left[2(a-b)+2(e^a-b)\cdot e^a\right]=0 \quad \text{and}$$

$$L_b=\frac{1}{2}(\sim)^{-\frac{1}{2}}\cdot\left[2(a-b)(-1)+2(e^a-b)(-1)\right]=0 \rightarrow$$

(*) $$a-b+(e^a-b)e^a=0 \\ b-a+(e^a-b)(-1)=0 \Big\} \quad \text{(add equations)} \rightarrow$$

$(e^a - b)(e^a - 1) = 0 \rightarrow \underline{\underline{\underline{b = e^a}}}$ or $\underline{\underline{\underline{e^a = 1}}}$;

if $\boxed{b = e^a}$ then equation

(*) $a - b + (e^a - b)e^a = 0$

becomes

$$a - e^a + (e^a - e^a)e^a = 0 \rightarrow$$

$$a - e^a = 0 \rightarrow a = e^a$$

(This is impossible! See graphs of $Y = X$ and $Y = e^X$.) ;

if $\boxed{e^a = 1}$ then $\boxed{a = 0}$ and equation

(*) $a - b + (e^a - b)e^a = 0$

becomes

$$0 - b + (1 - b)(1) = 0 \rightarrow$$

$1 - 2b = 0$ $\boxed{b = \frac{1}{2}}$; thus

$a = 0$ determines the point $(0, 1)$ on $Y = e^X$ and
$b = \frac{1}{2}$ determines the point $(\frac{1}{2}, \frac{1}{2})$ on $Y = X$

and the minimum distance is

$$\boxed{L = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = \frac{1}{\sqrt{2}}}\ .$$