Section 7.6

2.) $f(x,y) = xy$ and $2x + y - 4 = 0$, then

$$F(x,y,\lambda) = xy - \lambda(2x + y - 4)$$
$$= xy - 2\lambda x - \lambda y + 4\lambda \rightarrow$$

$$\begin{cases} F_x = y - 2\lambda = 0 \rightarrow \lambda = \frac{1}{2}y \\ F_y = x - \lambda = 0 \rightarrow \lambda = x \\ F_\lambda = -2x - y + 4 = 0 \end{cases} \rightarrow x = \frac{1}{2}y$$

$\rightarrow -2(\frac{1}{2}y) - y + 4 = 0 \rightarrow -2y + 4 = 0 \rightarrow$

$y = 2$, $x = 1$, $\lambda = 1$ and $f(1,2) = 2$ is maximum value of f.

4.) $f(x,y) = x^2 + y^2$ and $-2x - 4y + 5 = 0$, then

$$F(x,y,\lambda) = (x^2 + y^2) - \lambda(-2x - 4y + 5)$$
$$= x^2 + y^2 + 2\lambda x + 4\lambda y - 5\lambda \rightarrow$$

$$\begin{cases} F_x = 2x + 2\lambda = 0 \rightarrow \lambda = -x \\ F_y = 2y + 4\lambda = 0 \rightarrow \lambda = \frac{-1}{2}y \\ F_\lambda = 2x + 4y - 5 = 0 \end{cases} \quad -x = \frac{-1}{2}y \rightarrow x = \frac{1}{2}y$$

$\rightarrow 2(\frac{1}{2}y) + 4y - 5 = 0 \rightarrow 5y - 5 = 0 \rightarrow$

$y = 1$, $x = 1/2$, $\lambda = -1/2$ and $f(\frac{1}{2}, 1) = \frac{1}{4} + 1 = \frac{5}{4}$ is minimum value of f.

5.) $f(x,y) = x^2 - y^2$ and $y - x^2 = 0$, then

$$F(x,y,\lambda) = (x^2 - y^2) - \lambda(y - x^2)$$
$$= x^2 - y^2 - \lambda y + \lambda x^2 \rightarrow$$

$$\begin{cases} F_x = 2x + 2\lambda x = 2x(1+\lambda) = 0 \rightarrow \underline{\underline{x=0}} \text{ or } \underline{\underline{\lambda = -1}}; \\ F_y = -2y - \lambda = 0 \\ F_\lambda = -y + x^2 = 0 \quad ; \end{cases}$$

case 1: If $x=0$, then $-y+(0)^2=0 \to y=0$ and $-2(0)-\lambda=0 \to \lambda=0$ and value $f(0,0)=\underline{0}$;

case 2: If $\lambda=-1$, then $-2y+1=0 \to y=\frac{1}{2}$ and $-\frac{1}{2}+x^2=0 \to x^2=\frac{1}{2} \to x=\frac{1}{\sqrt{2}}$ or (NO) $\cancel{x=\frac{-1}{\sqrt{2}}}$; the maximum value

$$f\left(\frac{1}{\sqrt{2}},\frac{1}{2}\right)=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

7.) $f(x,y)=3x+xy+3y$ and $x+y-25=0$, then

$$F(x,y,\lambda)=(3x+xy+3y)-\lambda(x+y-25)$$
$$=3x+xy+3y-\lambda x-\lambda y+25\lambda \to$$

$$\begin{cases} F_x = 3+y-\lambda=0 \to \lambda=3+y \\ F_y = x+3-\lambda=0 \to \lambda=x+3 \\ F_\lambda = -x-y+25=0 \to \end{cases}$$

$3+y=x+3 \to y=x$

$\to -x-x+25=0 \to -2x+25=0 \to$

$x=\frac{25}{2}$, $y=\frac{25}{2}$, $\lambda=\frac{31}{2}$ and value

$f\left(\frac{25}{2},\frac{25}{2}\right)=\frac{75}{2}+\frac{625}{4}+\frac{75}{2}=\frac{925}{4}$ is a maximum value.

10.) $f(x,y)=\sqrt{x^2+y^2}$ and $2x+4y-15=0$, then

$$F(x,y,\lambda)=\sqrt{x^2+y^2}-\lambda(2x+4y-15)$$
$$=\sqrt{x^2+y^2}-2\lambda x-4\lambda y+15\lambda \to$$

$$\begin{cases} F_x = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}(\cancel{2}x)-2\lambda=0 \to \lambda=\frac{1}{2}\frac{x}{\sqrt{x^2+y^2}} \\ F_y = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cdot(\cancel{2}y)-4\lambda=0 \to \lambda=\frac{1}{4}\frac{y}{\sqrt{x^2+y^2}} \\ F_\lambda = -2x-4y+15=0 \end{cases}$$

$\frac{1}{2}x=\frac{1}{4}y \to y=2x$

$\to -2x-4(2x)+15=0$

$\to -10x+15=0 \to 10x=15 \to x=\frac{3}{2}$, $y=3$,

$$\lambda = \frac{1}{2}\,\frac{3/2}{\sqrt{\frac{9}{4}+9}} = \frac{1}{2}\,\frac{3/2}{\sqrt{\frac{45}{4}}} = \frac{1}{\cancel{2}}\,\frac{3/2}{\frac{3\sqrt{5}}{\cancel{2}}} = \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}\sqrt{5}} = \frac{1}{2\sqrt{5}},$$

and minimum value $f\left(\frac{3}{2}, 3\right) = \sqrt{\frac{45}{4}} = \frac{3}{2}\sqrt{5}$.

13.) $f(x,y,z) = 2x^2 + 3y^2 + 2z^2$ and $x+y+z-24=0$,
then $F(x,y,z,\lambda) = (2x^2+3y^2+2z^2) - \lambda(x+y+z-24)$
$= 2x^2 + 3y^2 + 2z^2 - \lambda x - \lambda y - \lambda z + 24\lambda \rightarrow$

$$\begin{cases} F_x = 4x - \lambda = 0 \rightarrow \lambda = 4x \\ F_y = 6y - \lambda = 0 \rightarrow \lambda = 6y \\ F_z = 4z - \lambda = 0 \rightarrow \lambda = 4z \\ F_\lambda = -x - y - z + 24 = 0 \end{cases}$$

$4x = 6y \rightarrow y = \frac{2}{3}x$

$6y = 4z \rightarrow z = \frac{3}{2}y = \frac{3}{2}\left(\frac{2}{3}x\right) \rightarrow z = x$

$\rightarrow -x - \left(\frac{2}{3}x\right) - (x) + 24 = 0 \rightarrow -\frac{8}{3}x + 24 = 0 \rightarrow$
$\frac{8}{3}x = 24 \rightarrow x = 9$, $y = 6$, $z = 9$, $\lambda = 36$,
and minimum value $f(9,6,9) = 432$.

14.) $f(x,y,z) = xyz$ and $x+y+z-6=0$, then
$F(x,y,z,\lambda) = xyz - \lambda(x+y+z-6)$
$= xyz - \lambda x - \lambda y - \lambda z + 6z \longrightarrow$

$$\begin{cases} F_x = yz - \lambda = 0 \rightarrow \lambda = yz \\ F_y = xz - \lambda = 0 \rightarrow \lambda = xz \\ F_z = xy - \lambda = 0 \rightarrow \lambda = xy \\ F_\lambda = -x - y - z + 6 = 0 \end{cases}$$

$yz = xz \rightarrow y = x$

$xz = xy \rightarrow z = y = x$

$\rightarrow -x - (x) - (x) + 6 = 0 \rightarrow -3x + 6 = 0 \rightarrow x = 2$,
$y = 2$, $z = 2$, $\lambda = 4$, and maximum value
$f(2,2,2) = 8$.

16.) $f(X,Y) = X^2 - 8X + Y^2 - 12Y + 48$ and $X+Y-8=0$, then

$F(X,Y,\lambda) = (X^2 - 8X + Y^2 - 12Y + 48) - \lambda(X+Y-8)$

$= X^2 - 8X + Y^2 - 12Y + 48 - \lambda X - \lambda Y + 8\lambda \rightarrow$

$F_X = 2X - 8 - \lambda = 0 \rightarrow \lambda = 2X - 8$ } $2X - 8 = 2Y - 12 \rightarrow$

$F_Y = 2Y - 12 - \lambda = 0 \rightarrow \lambda = 2Y - 12$ } $2Y = 2X + 4 \rightarrow$

$F_\lambda = -X - Y + 8 = 0$ $\leftarrow Y = X + 2$

$\rightarrow -X - (X+2) + 8 = 0 \rightarrow -2X + 6 = 0 \rightarrow$

$X = 3$, $Y = 5$, $\lambda = -2$, and minimum value $f(3,5) = -2$.