Section C.2

1.) $\frac{dy}{dx} = \frac{x}{y+3} \rightarrow (y+3)\,dy = x\,dx$ (YES)

4.) $\frac{dy}{dx} = \frac{x}{x+y} \rightarrow (x+y)\,dy = x\,dx$ (NO)

6.) $x\,\frac{dy}{dx} = \frac{1}{y} \rightarrow y\,dy = \frac{1}{x}\,dx$ (YES)

7.) $\frac{dy}{dx} = 2x \rightarrow \int dy = \int 2x\,dx \rightarrow$
$y = x^2 + c$.

* 10.) $\frac{dy}{dx} = x^2 y \rightarrow \int \frac{1}{y}\,dy = \int x^2\,dx \rightarrow$
$\ln|y| = \frac{1}{3}x^3 + c$. and $y=0$!

12.) $(1+y)\frac{dy}{dx} - 4x = 0 \rightarrow (1+y)\frac{dy}{dx} = 4x \rightarrow$
$\int (1+y)\,dy = \int 4x\,dx \rightarrow$
$y + \frac{1}{2}y^2 = 2x^2 + c$

* 14.) $y' - y = 5 \rightarrow \frac{dy}{dx} = y+5 \rightarrow$
$\int \frac{1}{y+5}\,dy = \int dx \rightarrow \ln|y+5| = x + c$ and $y=-5$!

15.) $\frac{dy}{dt} = \frac{e^t}{4y} \rightarrow \int 4y\,dy = \int e^t\,dt \rightarrow$
$2y^2 = e^t + c$

* 17.) $\frac{dy}{dx} = \sqrt{1-y} \to \int \frac{1}{\sqrt{1-y}}\,dy = \int dx \to$

$\int (1-y)^{-1/2}dy = x+c \to -1\cdot\frac{(1-y)^{1/2}}{1/2} = x+c \to$

$-2(1-y)^{1/2} = x+c$ and $y=1$!

18.) $\frac{dy}{dx} = \frac{\sqrt{x}}{\sqrt{y}} \to \int \sqrt{y}\,dy = \int \sqrt{x}\,dx \to$

$\int y^{1/2}dy = \int x^{1/2}dx \to \frac{y^{3/2}}{3/2} = \frac{x^{3/2}}{3/2} + c \to$

$\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + c$.

* 20.) $\frac{dy}{dx} = (2x-1)(y+3) \to \int \frac{1}{y+3}\,dy = \int (2x-1)\,dx$

$\to \ln|y+3| = x^2 - x + c$ and $y=-3$!

23.) $y' = \frac{x}{y} - \frac{x}{1+y} \to \frac{dy}{dx} = \left(\frac{1}{y} - \frac{1}{1+y}\right)x \to$

$dy = \frac{1+\cancel{y}-\cancel{y}}{y(1+y)}\cdot x\,dx \to \int (y+y^2)\,dy = \int x\,dx \to$

$\frac{1}{2}y^2 + \frac{1}{3}y^3 = \frac{1}{2}x^2 + c$.

25.) $e^x(y'+1) = 1 \to y'+1 = \frac{1}{e^x} \to$

$\frac{dy}{dx} = e^{-x} - 1 \to \int dy = \int (e^{-x}-1)\,dx \to$

$y = -e^{-x} - x + c$.

26.) $yy' - 2xe^x = 0 \rightarrow y \cdot \frac{dy}{dx} = 2xe^x \rightarrow$

$\int y\,dy = \int 2xe^x\,dx$ (Let $u = 2x$, $dv = e^x dx$, $du = 2\,dx$, $v = e^x$)

$\rightarrow \frac{1}{2}y^2 = 2xe^x - 2\int e^x dx \rightarrow$

$\frac{1}{2}y^2 = 2xe^x - 2(e^x + c)$.

27.) $yy' - e^x = 0 \rightarrow yy' = e^x \rightarrow$

$y\frac{dy}{dx} = e^x \rightarrow \int y\,dy = \int e^x dx \rightarrow$

$\frac{1}{2}y^2 = e^x + c$ and $x = 0$, $y = 4 \rightarrow$

$\frac{1}{2}(4)^2 = e^0 + c \rightarrow 8 = 1 + c \rightarrow c = 7 \rightarrow$

$\frac{1}{2}y^2 = e^x + 7$.

28.) $\sqrt{x} + \sqrt{y}\,y' = 0 \rightarrow y^{1/2} \cdot \frac{dy}{dx} = -x^{1/2} \rightarrow$

$\int y^{1/2}\,dy = \int -x^{1/2}\,dx \rightarrow$

$\frac{y^{3/2}}{3/2} = \frac{-x^{3/2}}{3/2} + c \rightarrow \frac{2}{3}y^{3/2} = -\frac{2}{3}x^{3/2} + c$

and $x = 1$, $y = 4 \rightarrow \frac{2}{3}(4)^{3/2} = -\frac{2}{3}(1)^{3/2} + c \rightarrow$

$\frac{2}{3}(8) = -\frac{2}{3}(1) + c \rightarrow c = \frac{18}{3} \rightarrow$

$\frac{2}{3}y^{3/2} = -\frac{2}{3}x^{3/2} + \frac{18}{3} \rightarrow$

$y^{3/2} = -x^{3/2} + 9$.

30.) $\frac{dy}{dx} = x^2(1+y) \rightarrow \int \frac{1}{1+y}\,dy = \int x^2\,dx$

$\rightarrow \underline{\ln|1+y| = \frac{1}{3}x^3 + c}$ and $x=0,\ y=3 \rightarrow$

$\ln 4 = 0 + c \rightarrow c = \ln 4 \rightarrow$

$\ln|1+y| = \frac{1}{3}x^3 + \ln 4$.

33.) $y' = \frac{6x}{5y} \rightarrow 5y\,\frac{dy}{dx} = 6x \rightarrow$

$\int 5y\,dy = \int 6x\,dx \rightarrow \underline{\frac{5}{2}y^2 = 3x^2 + c}$ and

$x=-1,\ y=1 \rightarrow \frac{5}{2}(1)^2 = 3(-1)^2 + c \rightarrow$

$\frac{5}{2} = 3 + c \rightarrow c = \frac{-1}{2} \rightarrow \frac{5}{2}y^2 = 3x^2 - \frac{1}{2}$.

34.) $y' = \frac{2y}{3x} \rightarrow \frac{1}{2y}\cdot\frac{dy}{dx} = \frac{1}{3x} \rightarrow$

$\frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{3}\int \frac{1}{x}\,dx \rightarrow$

$\underline{\frac{1}{2}\ln|y| = \frac{1}{3}\ln|x| + c}$ and $x=8,\ y=2$

$\rightarrow \frac{1}{2}\ln 2 = \frac{1}{3}\ln 8 + c \rightarrow \frac{1}{2}\ln 2 = \frac{1}{3}\ln 2^3 + c$

$\rightarrow \frac{1}{2}\ln 2 = \frac{1}{3}\cdot 3\ln 2 + c \rightarrow$

$\frac{1}{2}\ln 2 = \ln 2 + c \rightarrow c = \frac{-1}{2}\ln 2 \rightarrow$

$\frac{1}{2}\ln|y| = \frac{1}{3}\ln|x| - \frac{1}{2}\ln 2$.

Newton's Law of Cooling : $\frac{dT}{dt} = k(T - T_0) \rightarrow$

$\int \frac{1}{T - T_0}\, dT = \int k\, dt \rightarrow \ln|T - T_0| = kt + c_1 \rightarrow$

$e^{\ln|T - T_0|} = e^{kt + c_1} \rightarrow |T - T_0| = e^{c_1} e^{kt} \rightarrow$

$|T - T_0| = c_2 e^{kt} \rightarrow T - T_0 = ce^{kt} \rightarrow$

$$\boxed{T = T_0 + ce^{kt}}$$

37.) $T = T_0 + ce^{kt}$ and $T_0 = 90°F$;

$t = 0$ hr., $T = 1500°F$ so

$1500 = 90 + ce^{k(0)} = 90 + c \rightarrow c = 1410$

$\rightarrow T = 90 + 1410\, e^{kt}$;

$t = 1$ hr., $T = 1120°F$ so

$1120 = 90 + 1410\, e^{k} \rightarrow 1030 = 1410 e^{k}$

$\rightarrow \frac{103}{141} = e^{k} \rightarrow \ln\left(\frac{103}{141}\right) = \ln e^{k} = k \rightarrow$

$T = 90 + 1410\, e^{\ln\left(\frac{103}{141}\right) \cdot t}$

$= 90 + 1410 \left[e^{\ln\left(\frac{103}{141}\right)}\right]^{t} \rightarrow$

$T = 90 + 1410 \left(\frac{103}{141}\right)^{t}$;

if $t = 5$ hrs. then

$T = 90 + 1410 \left(\frac{103}{141}\right)^{5} \approx 383.3\ °F$.

39.) $T = T_0 + ce^{kt}$ and $T_0 = 0°F$;

$t = 0$ hr., $T = 70°F$ so $70 = 0 + ce^0 = c \to c = 70 \to$

$T = 70e^{kt}$;

$t = 1$ hr., $T = 48°F$ so $48 = 70e^{k(1)} \to$

$\frac{48}{70} = e^k \to \ln\left(\frac{24}{35}\right) = \ln e^k = k \to$

$$T = 70\, e^{t \cdot \ln\left(\frac{24}{35}\right)}$$

$$= 70\left[e^{\ln\left(\frac{24}{35}\right)}\right]^t \longrightarrow$$

$$T = 70\left(\frac{24}{35}\right)^t \quad ;$$

a.) if $t = 6$ hrs., then $T = 70\left(\frac{24}{35}\right)^6 = 7.3°F$;

b.) if $T = 10°F$, then

$10 = 70\left(\frac{24}{35}\right)^t \to \frac{1}{7} = \left(\frac{24}{35}\right)^t \to \ln\left(\frac{1}{7}\right) = t \ln\left(\frac{24}{35}\right)$

$\to t = \dfrac{\ln\left(\frac{1}{7}\right)}{\ln\left(\frac{24}{35}\right)} = 5.16$ hrs. .

40.) $\frac{dV}{dt} = kV^{2/3} \to \int \frac{1}{V^{2/3}}\, dV = \int k\, dt \to$

$\int V^{-2/3}\, dV = kt + c \to$

$\frac{V^{1/3}}{1/3} = kt + c \to 3V^{1/3} = kt + c \to$

$V^{1/3} = \frac{kt + c}{3} \to V = \left(\frac{kt + c}{3}\right)^3 \to$

$$V = \frac{1}{27}(kt + c)^3 \quad .$$

42.) $\frac{dS}{dt} = \frac{kS}{t^2} \rightarrow \int \frac{1}{S}\, dS = \int \frac{k}{t^2}\, dt$

$\rightarrow \ln|S| = -\frac{k}{t} + c$ ($S > 0$ so $|S| = S$)

$\rightarrow \ln S = \frac{-k}{t} + c$

$\rightarrow S = e^{-\frac{k}{t} + c}$; if $\lim_{t \to \infty} S = 50$ then

$\lim_{t \to \infty} \left(e^{\frac{-k}{t} + c} \right) = e^{0+c} = e^{c} = 50 \rightarrow c = \ln 50$

so $S = e^{\frac{-k}{t} + \ln 50}$;

when $t = 1$ yr., $S = 10$ (10,000 units) so

$10 = e^{-k + \ln 50} \rightarrow \ln 10 = -k + \ln 50 \rightarrow$

$k = \ln 50 - \ln 10 = \ln\left(\frac{50}{10}\right) = \ln 5$; then

$S = e^{\frac{-\ln 5}{t} + \ln 50} = e^{\ln 50} \cdot e^{-\frac{\ln 5}{t}}$

$= 50 \cdot e^{\ln 5^{-1} \cdot 1/t}$

$= 50 \left(e^{\ln(1/5)} \right)^{1/t}$

$= 50 \left(\frac{1}{5}\right)^{1/t} \longrightarrow S = 50\left(\frac{1}{5}\right)^{1/t}$