Section 10.6

4.) $f(x) = x^5 + x - 1 \rightarrow f'(x) = 5x^4 + 1$ so

$$x_{n+1} = x_n - \frac{x_n^5 + x_n - 1}{5x_n^4 + 1}$$: let $x_0 = 1$ then

$$x_1 = x_0 - \frac{x_0^5 + x_0 - 1}{5x_0^4 + 1} = 5/6 = .833333333,$$

$$x_2 = x_1 - \frac{x_1^5 + x_1 - 1}{5x_1^4 + 1} = .764382115,$$

$$x_3 = x_2 - \frac{x_2^5 + x_2 - 1}{5x_2^4 + 1} = .755024867,$$

$$x_4 = x_3 - \frac{x_3^5 + x_3 - 1}{5x_3^4 + 1} = .754877701$$ so

$$r \approx .755$$

9.) $f(x) = e^{-x^2} - x^2 \rightarrow f'(x) = -2xe^{-x^2} - 2x$ so

$$x_{n+1} = x_n - \frac{e^{-x_n^2} - x_n^2}{-2x_n e^{-x_n^2} - 2x_n} \overset{\text{algebra}}{=} \cdots = \frac{x_n^2(2 + e^{x_n^2}) + 1}{2x_n(1 + e^{x_n^2})};$$

Find r_1 : let $x_0 = -.75$ then

$$x_1 = \frac{x_0^2(2 + e^{x_0^2}) + 1}{2x_0(1 + e^{x_0^2})} = -.753092922,$$

$$x_2 = \frac{x_1^2(2 + e^{x_1^2}) + 1}{2x_1(1 + e^{x_1^2})} = -.753089165$$ so

$$r_1 \approx -.75309$$

Find r_2: let $x_0 = .75$ then

$$x_1 = \frac{x_0^2(2+e^{x_0^2})+1}{2x_0(1+e^{x_0^2})} = .753092922,$$

$$x_2 = \frac{x_1^2(2+e^{x_1^2})+1}{2x_1(1+e^{x_1^2})} = .753089165 \quad \text{so}$$

$$\boxed{r_2 \approx .75309}.$$

14.) $2x+1 = \sqrt{x+4} \rightarrow$ solve

$F(x) = 2x+1-\sqrt{x+4} = 0 \rightarrow F'(x) = 2 - \frac{1}{2}(x+4)^{-1/2}$

then

$$x_{n+1} = x_n - \frac{2x_n+1-\sqrt{x_n+4}}{2-\frac{1}{2}(x+4)^{-1/2}} \quad : \quad \text{let } x_0 = .5$$

then

$$x_1 = x_0 - \frac{2x_0+1-\sqrt{x_0+4}}{2-\frac{1}{2}(x_0+4)^{-1/2}} = .568764098,$$

$$x_2 = x_1 - \frac{2x_1+1-\sqrt{x_1+4}}{2-\frac{1}{2}(x_1+4)^{-1/2}} = .568729304 \quad \text{so}$$

$$\boxed{r \approx .5687}.$$

16.) $x = e^{-x} \rightarrow$ solve $F(x) = x - e^{-x} = 0 \rightarrow$

$F'(x) = 1 + e^{-x}$ and

algebra

$$x_{n+1} = x_n - \frac{x_n - e^{-x_n}}{1+e^{-x_n}} = \cdots = \frac{x_n+1}{e^{x_n}+1} \; : \; \text{let } x_0 = \frac{2}{3}$$

then

$$x_1 = \frac{x_0+1}{e^{x_0}+1} = .565406052,$$

$$x_2 = \frac{x_1+1}{e^{x_1}+1} = .567142744 ,$$

$$x_3 = \frac{x_2+1}{e^{x_2}+1} = .56714329 \quad \text{so}$$

$$\boxed{r \approx .56714}$$

31.) $f(x) = -x^3 + 3x^2 - x + 1 \rightarrow f'(x) = -3x^2 + 6x - 1$ and

$$x_{n+1} = x_n - \frac{-x_n^3 + 3x_n^2 - x_n + 1}{-3x_n^2 + 6x_n - 1} = \cdots = \frac{-2x_n^3 + 3x_n^2 - 1}{-3x_n^2 + 6x_n - 1} :$$

Let $x_0 = 1$ then

$$x_1 = \frac{-2x_0^3 + 3x_0^2 - 1}{-3x_0^2 + 6x_0 - 1} = 0 ,$$

$$x_2 = \frac{-2x_1^3 + 3x_1^2 - 1}{-3x_1^2 + 6x_1 - 1} = 1 ,$$

$x_3 = 0$, $x_4 = 1$, etc. (oscillates repeatedly)

36.) Solve $f(x) = x^2 - 5 = 0 \rightarrow f'(x) = 2x$ so

$$x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n} = \frac{x_n^2 + 5}{2x_n} : \text{ let } x_0 = 2 \text{ then}$$

$$x_1 = \frac{x_0^2 + 5}{2x_0} = \frac{9}{4} = 2.25, \quad x_2 = \frac{x_1^2 + 5}{2x_1} = 2.23611111 ,$$

$$x_3 = \frac{x_2^2 + 5}{2x_2} = 2.236067978 \quad \text{so} \quad \boxed{\sqrt{5} \approx 2.236} .$$

39.) If $x = \frac{1}{a}$ then $\frac{1}{x} = a$ and

$f(x) = \frac{1}{x} - a = 0$ so Newton's method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\frac{1}{x_n} - a}{\frac{-1}{x_n^2}} = x_n + x_n - a x_n^2$$

$$= 2x_n - a x_n^2 = x_n(2 - a x_n) \text{ or } \boxed{x_{n+1} = x_n(2 - a x_n)} .$$

44.) Find the minimum value of

$C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right)$ for $x \geq 1$:

$$C' = 100\left(\frac{-400}{x^3} + \frac{30}{(x+30)^2}\right) = 0 \rightarrow \frac{-40}{x^3} + \frac{3}{(x+30)^2} = 0 \rightarrow$$

$$\frac{-40(x+30)^2 + 3x^3}{x^3(x+30)^2} = 0 \rightarrow -40(x^2 + 60x + 900) + 3x^3 = 0 \rightarrow$$

$$f(x) = 3x^3 - 40x^2 - 2400x - 36{,}000 = 0 \rightarrow$$

Estimate the root of $f(x) = 0$ and then apply Newton's method :

$f(1) = -38{,}437$
$f(10) = -61{,}000$
$f(20) = -76{,}000$
$f(30) = -63{,}000$

$f(40) = -4000$
$f(50) = 119{,}000$

These numbers show that the solution to $f(x) = 0$ is between $x = 40$ and $x = 50$.

NEWTON : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \rightarrow$

$$x_{n+1} = x_n - \frac{3x_n^3 - 40x_n^2 - 2400x_n - 36{,}000}{90x_n^2 - 80x_n - 2400} \rightarrow$$

$$\boxed{x_{n+1} = \frac{6x_n^3 - 40x_n^2 + 36{,}000}{9x_n^2 - 80x_n - 2400}} \; ;$$

Let $x_1 = 40$ then

$x_2 = 40.45454545$,
$x_3 = 40.44724323$,
$x_4 = 40.46830656$,
$x_5 = 40.44725713$,
$x_6 = 40.44724133$. Thus,

the minimum cost $C = 69.64$ ($= \$69{,}640$) occurs when

$x = 40.44$ ($= 4044$ orders).