

Section 7.3

$$2.) \quad f(x, y) = 4 - x^2 - 4y^2$$

$$a.) \quad f(0, 0) = 4 - 0 - 0 = 4$$

$$b.) \quad f(0, 1) = 4 - 0 - 4 = 0$$

$$c.) \quad f(2, 3) = 4 - 4 - 36 = -36$$

$$d.) \quad f(1, y) = 4 - 1 - 4y^2 = 3 - 4y^2$$

$$e.) \quad f(x, 0) = 4 - x^2 - 0 = 4 - x^2$$

$$f.) \quad f(t, 1) = 4 - t^2 - 4 = -t^2$$

$$3.) \quad f(x, y) = xe^y$$

$$a.) \quad f(5, 0) = 5e^0 = 5$$

$$b.) \quad f(3, 2) = 3e^2$$

$$c.) \quad f(2, -1) = 2e^{-1}$$

$$d.) \quad f(5, y) = 5e^y$$

$$e.) \quad f(x, 2) = xe^2$$

$$f.) \quad f(t, t) = te^t$$

$$6.) \quad f(x, y, z) = \sqrt{x+y+z}$$

$$a.) \quad f(0, 5, 4) = \sqrt{9} = 3$$

$$b.) \quad f(6, 8, -3) = \sqrt{11}$$

$$11.) \quad f(x, y) = \int_x^y (2t - 3) dt = (t^2 - 3t) \Big|_x^y$$

$$= (y^2 - 3y) - (x^2 - 3x)$$

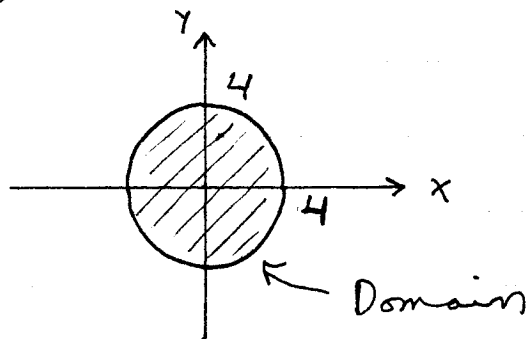
$$= y^2 - 3y - x^2 + 3x$$

$$a.) \quad f(1, 2) = 4 - 6 - 1 + 3 = 0$$

$$b.) \quad f(1, 4) = 16 - 12 - 1 + 3 = 6$$

15.) $f(x,y) = \sqrt{16-x^2-y^2}$, $16-x^2-y^2 \geq 0 \rightarrow$

$x^2+y^2 \leq 16$ so



Domain: all (x,y)

satisfying $x^2+y^2 \leq 4^2$
(circle and interior)

Range: all values $z \geq 0$ satisfying

$z = \sqrt{16-x^2-y^2} \rightarrow z^2 = 16-x^2-y^2 \rightarrow$

$x^2+y^2+z^2 = 4^2$ (top $\frac{1}{2}$ of sphere),

so range is all z -values
satisfying $0 \leq z \leq 4$.

16.) $f(x,y) = x^2+y^2-1$,

Domain: all points (x,y) ;

Range: $z = x^2+y^2-1 = (x^2+y^2)-1$;

since $x^2+y^2 \geq 0$, the range is
all values $z \geq -1$.

17.) $f(x,y) = e^{\frac{x}{y}}$, $y \neq 0$ so

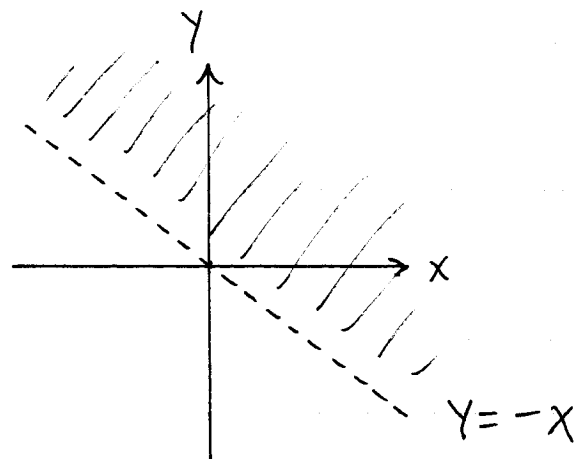
Domain: all (x,y) with $y \neq 0$ (everything

in 2-space except the x-axis);
Range: $z = e^{x/y} > 0$ and for fixed $y (> 0)$

$\lim_{x \rightarrow +\infty} e^{x/y} = +\infty$ and $\lim_{x \rightarrow -\infty} e^{x/y} = 0$.
values $(z > 0)$. so range is all

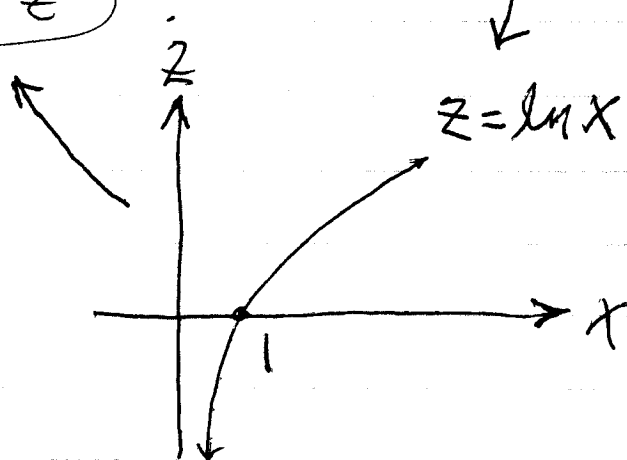
18.) $f(x, y) = \ln(x+y)$, $x+y > 0 \rightarrow y > -x$ so

Domain: all (x, y) with
 $y > -x$ (above
line $y = -x$)



Range: If we let $y=0$,

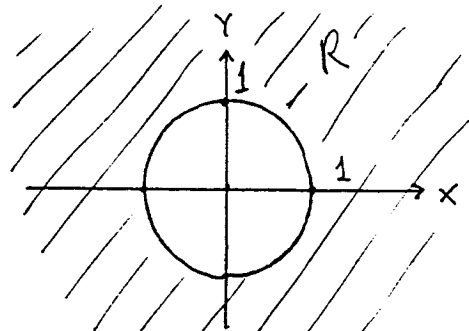
then $z = \ln(x+y) = \ln(x+0) = \ln x$ assumes
all real values, so range is
all real numbers z



20.) $f(x,y) = \sqrt{x^2+y^2-1}$ then

$x^2+y^2-1 \geq 0 \rightarrow x^2+y^2 \geq 1$ (a circle and its exterior) so domain is all points (x,y) satisfying $x^2+y^2 \geq 1$:

Range : all $z \geq 0$.



22.) $f(x,y) = \frac{4y}{x-1}$ then

$x \neq 1$ so domain is all points

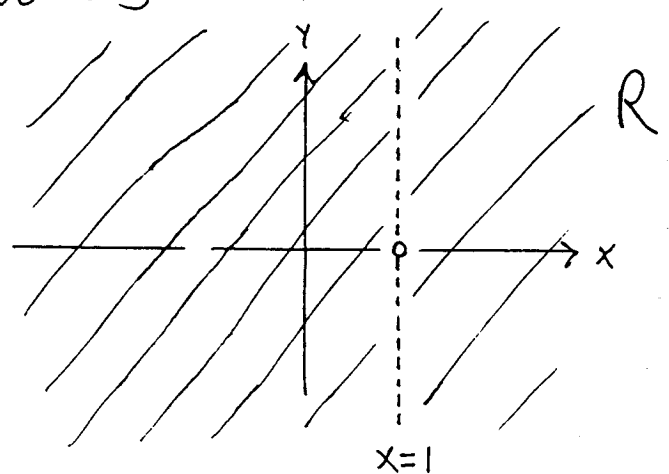
(x,y) satisfying

$x \neq 1$:

(domain is all points

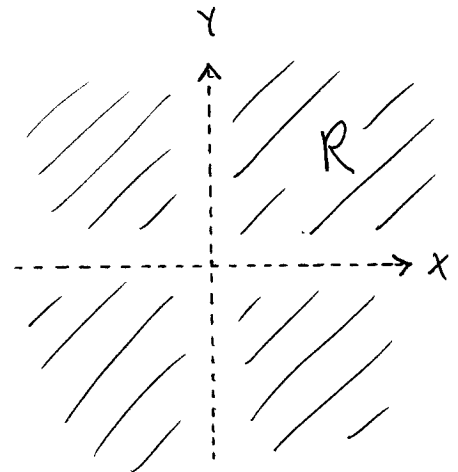
(x,y) not lying on the line $x=1$);

Range : all values of z .



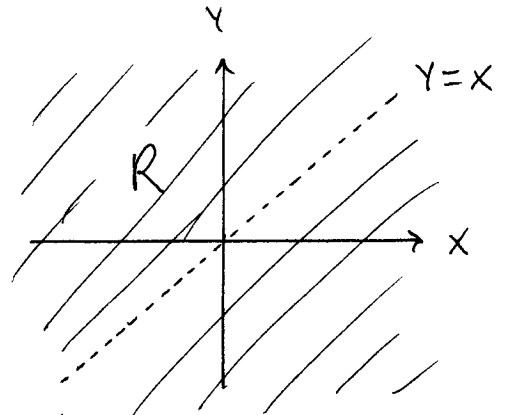
23.) $f(x,y) = \frac{1}{xy}$ then
 $x \neq 0$ or $y \neq 0$ so domain
 is all points (x,y) with
 $x \neq 0$ or $y \neq 0$:

(domain is all points
 (x,y) not lying on x -axis
 or y -axis); Range: all $z \neq 0$.

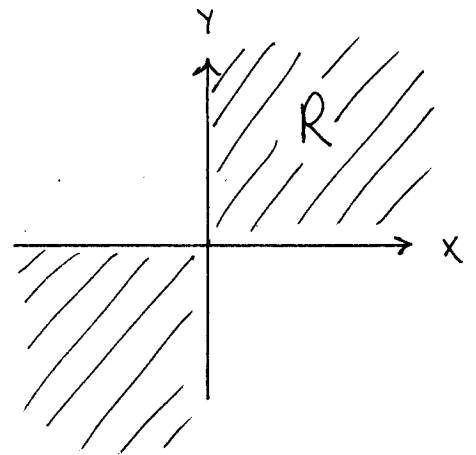


24.) $g(x,y) = \frac{1}{x-y}$ then
 $x-y \neq 0$ or $y \neq x$ so domain
 is all points (x,y) with

$y \neq x$: (domain is all
 points (x,y) not lying
 on line $y=x$); Range: all $z \neq 0$.



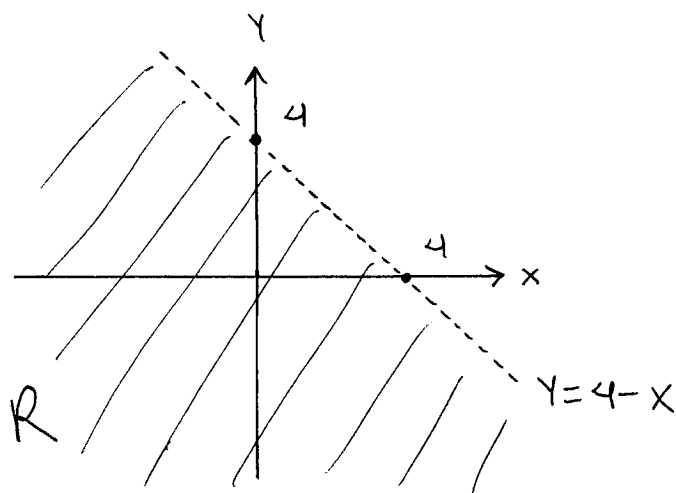
26.) $f(x,y) = \sqrt{xy}$ then
 $xy \geq 0$ so $x \geq 0$ and $y \geq 0$
 or $x \leq 0$ and $y \leq 0$ so domain
 is all points (x,y) on x -axis
 or y -axis or in 1st or 3rd
 quadrants; Range: all $z \geq 0$.



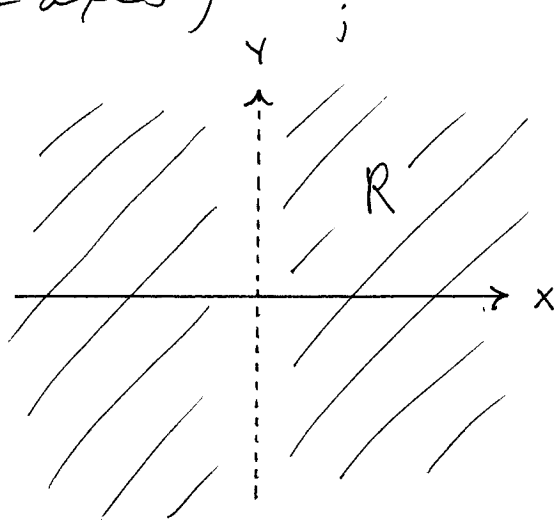
27.) $g(x,y) = \ln(4-x-y)$ then
 $4-x-y > 0 \rightarrow 4-x > y$ so domain
 is all points (x,y) satisfying $y < 4-x$:
 (domain is all points (x,y) lying below

the line
($y = 4 - x$)

Range: all
values of z .



28.) $f(x, y) = ye^{\frac{1}{x}}$ then $x \neq 0$ so
domain is all points (x, y) with $x \neq 0$:
(domain is all points (x, y) not lying
on the y -axis)



Range: all values of z .

46.)

$$W(x, y) = \frac{1}{x-y}$$

a.) $W(15, 10) = \frac{1}{5} \text{ hr.} = 12 \text{ min.}$

b.) $W(12, 9) = \frac{1}{3} \text{ hr.} = 20 \text{ min.}$

c.) $W(12, 6) = \frac{1}{6} \text{ hr.} = 10 \text{ min.}$

d.) $W(4, 2) = \frac{1}{2} \text{ hr.} = 30 \text{ min.}$