

Section 10.1

$$9.) \lim_{n \rightarrow \infty} \frac{5}{n} = \frac{5}{\infty} = 0$$

$$10.) \lim_{n \rightarrow \infty} \frac{n}{2} = \frac{\infty}{2} = \infty \text{ (diverges)}$$

$$11.) \lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = 1 + 0 = 1$$

$$12.) \lim_{n \rightarrow \infty} \frac{1}{n^{3/2}} = \frac{1}{\infty} = 0$$

$$13.) \lim_{n \rightarrow \infty} \frac{n^2 + 3n - 4}{2n^2 + n - 3} = \lim_{n \rightarrow \infty} \frac{1 + \frac{3}{n} - \frac{4}{n^2}}{2 + \frac{1}{n} - \frac{3}{n^2}}$$

$$= \frac{1 + 0 - 0}{2 + 0 - 0} = \frac{1}{2}$$

$$14.) \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{1+1/n}}$$

$$= \sqrt{\frac{1}{1+0}} = \sqrt{1} = 1$$

$$15.) \lim_{n \rightarrow \infty} \frac{n^2 - 25}{n+5} = \lim_{n \rightarrow \infty} \frac{(n-5)(\cancel{n+5})}{\cancel{n+5}}$$

$$= \infty \text{ (diverges)}$$

$$16.) \lim_{n \rightarrow \infty} \frac{n+2}{n^2+1} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{2}{n^2}}{1 + \frac{1}{n^2}} = \frac{0+0}{1+0} = 0$$

$$17.) 0 \leq \frac{1+(-1)^n}{n} \leq \frac{2}{n} \text{ and } \lim_{n \rightarrow \infty} \frac{2}{n} = 0$$

$$\text{so } \lim_{n \rightarrow \infty} \frac{1+(-1)^n}{n} = 0 \text{ by "squeeze"}$$

$$18.) a_n = 1+(-1)^n : 0, 2, 0, 2, 0, 2, \dots$$

position : 1 2 3 4 5 6 ...

$\lim_{n \rightarrow \infty} (1 + (-1)^n)$ DNE by oscillation

$$19.) \lim_{n \rightarrow \infty} \frac{n!}{n} = \lim_{n \rightarrow \infty} \frac{\cancel{n}(n-1)(n-2) \cdots (3)(2)(1)}{\cancel{n}}$$
$$= \infty \text{ (diverges)}$$

$$20.) \lim_{n \rightarrow \infty} \frac{n!}{(n+1)!} = \lim_{n \rightarrow \infty} \frac{\cancel{n}(n+1)\cancel{(n-2)} \cdots (3)(2)(1)}{(n+1)\cancel{n}(n+1)\cancel{(n-2)} \cdots (3)(2)(1)}$$
$$= \lim_{n \rightarrow \infty} \frac{1}{n+1} = \frac{1}{\infty} = 0$$

$$21.) \lim_{n \rightarrow \infty} \left(3 - \frac{1}{2^n}\right) = 3 - \frac{1}{\infty} = 3 - 0 = 3$$

$$22.) \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n \rightarrow \infty} \frac{\cancel{n}}{\sqrt{\cancel{n}^2(1 + \frac{1}{n^2})}}$$
$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+0}} = \frac{1}{\sqrt{1+0}} = 1$$

$$23.) \lim_{n \rightarrow \infty} \frac{3^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0 \text{ since}$$
$$-1 < \frac{3}{4} < 1$$

$$24.) \lim_{n \rightarrow \infty} (0.5)^n = 0 \text{ since } -1 < 0.5 < 1$$

$$25.) \lim_{n \rightarrow \infty} \frac{(n+1)!}{n!} = \lim_{n \rightarrow \infty} \frac{(n+1)\cancel{n}(n-1)(n-2) \cdots (3)(2)(1)}{n\cancel{(n-1)}(n-2) \cdots (3)(2)(1)}$$
$$= \lim_{n \rightarrow \infty} (n+1) = \infty \text{ (diverges)}$$

$$26.) \lim_{n \rightarrow \infty} \frac{(n-2)!}{n!} = \lim_{n \rightarrow \infty} \frac{\cancel{(n-2)}(n-3) \cdots (3)(2)(1)}{n(n-1)\cancel{(n-2)}(n-3) \cdots (3)(2)(1)}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n(n-1)} = \frac{1}{\infty} = 0$$

$$27.) \lim_{n \rightarrow \infty} \frac{n}{n+1} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} = \frac{1}{1+0} = 1$$

so $\lim_{n \rightarrow \infty} (-1)^n \cdot \frac{n}{n+1}$ diverges by oscillation

$$28.) \lim_{n \rightarrow \infty} \left(\frac{n-1}{n} - \frac{n}{n-1} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{(n-1)^2 - n^2}{n(n-1)} = \lim_{n \rightarrow \infty} \frac{\cancel{n^2} - 2n + 1 - \cancel{n^2}}{n^2 - n}$$

$$= \lim_{n \rightarrow \infty} \frac{-2 - \frac{1}{n}}{n-1} = \frac{-2-0}{\infty} = 0$$

Section 10.2

$$6.) \quad \lim_{n \rightarrow \infty} \frac{n}{2n+3} = \lim_{n \rightarrow \infty} \frac{1}{2 + \frac{3}{n}} = \frac{1}{2+0} = \frac{1}{2} \neq 0$$

$$\text{so } \sum_{n=1}^{\infty} \frac{n}{2n+3} \text{ diverges.}$$

$$7.) \quad \lim_{n \rightarrow \infty} \frac{n^2}{n^2+1} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n^2}} = \frac{1}{1+0} = 1 \neq 0$$

$$\text{so } \sum_{n=1}^{\infty} \frac{n^2}{n^2+1} \text{ diverges.}$$

$$8.) \quad \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n^2}{n^2+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{1 + \frac{1}{n^2}}} = \sqrt{1} = 1 \neq 0$$

$$\text{so } \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} \text{ diverges.}$$

$$9.) \quad \lim_{n \rightarrow \infty} 3 \left(\frac{3}{2}\right)^n = \infty \neq 0 \quad \text{so}$$

$$\sum_{n=0}^{\infty} 3 \left(\frac{3}{2}\right)^n \text{ diverges}$$

$$18.) \quad \sum_{n=0}^{\infty} 2 \left(\frac{2}{3}\right)^n = 2 \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = 2 \left(1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots\right) \\ = 2 \cdot \frac{1}{1 - \left(\frac{2}{3}\right)} = 2 \cdot 3 = \textcircled{6}$$

$$19.) \quad \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots = \frac{1}{1 - \left(-\frac{1}{2}\right)} = \textcircled{\frac{2}{3}}$$

$$24.) \quad 8 + 6 + \frac{9}{2} + \frac{27}{8} + \dots = 8 \left[1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \dots\right] \\ = 8 \left[1 + \left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots\right] = 8 \cdot \frac{1}{1 - \left(\frac{3}{4}\right)} = 8 \cdot 4 = \textcircled{32}$$

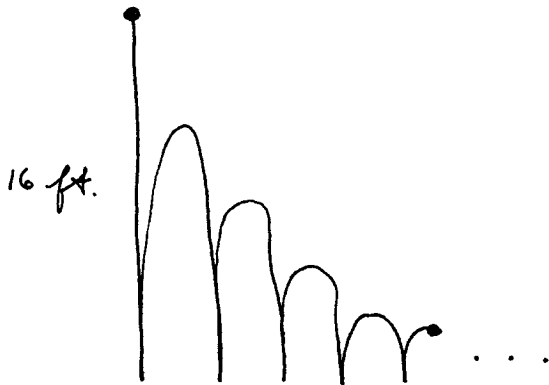
$$26.) \quad 4 - 2 + 1 - \frac{1}{2} + \dots = 4 \left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots \right]$$

$$= 4 \left[1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots \right] = 4 \cdot \frac{1}{1 - \left(-\frac{1}{2}\right)} = 4 \cdot \frac{2}{3} = \left(\frac{8}{3}\right)$$

$$28.) \quad \sum_{n=0}^{\infty} \left[(0.7)^n + (0.9)^n \right] = \sum_{n=0}^{\infty} (0.7)^n + \sum_{n=0}^{\infty} (0.9)^n$$

$$= \frac{1}{1 - (0.7)} + \frac{1}{1 - (0.9)} = \frac{10}{3} + \frac{10}{1} = \left(\frac{40}{3}\right)$$

47.)



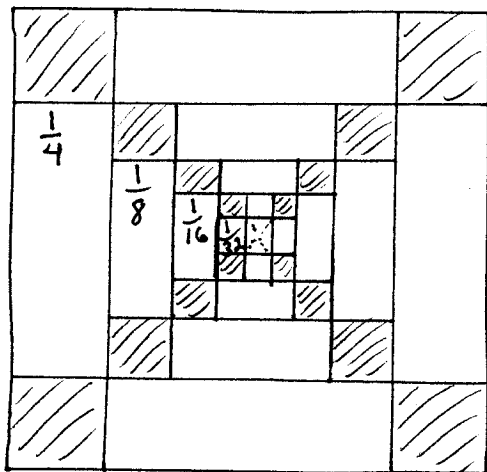
Total distance traveled is

$$T = 16 + (0.64)(16)(2) + (0.64)^2(16)(2) + (0.64)^3(16)(2) + \dots$$

$$= 16 + (0.64)(16)(2) \left[1 + (0.64) + (0.64)^2 + (0.64)^3 + \dots \right]$$

$$= 16 + (20.48) \frac{1}{1 - (0.64)} \approx \left(72.89 \text{ ft.}\right)$$

52.)



Assume large square is 1×1 then total shaded area is

$$\begin{aligned}
 T &= 4 \cdot \left(\frac{1}{4}\right)^2 + 4 \left(\frac{1}{8}\right)^2 + 4 \left(\frac{1}{16}\right)^2 + 4 \left(\frac{1}{32}\right)^2 + \dots \\
 &= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots \\
 &= \frac{1}{4} \left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots \right] \\
 &= \frac{1}{4} \cdot \frac{1}{1 - \left(\frac{1}{4}\right)} = \frac{1}{4} \cdot \frac{4}{3} = \left(\frac{1}{3}\right) \text{ of total area.}
 \end{aligned}$$

51.) Total \$ spent is (in millions)

$$\begin{aligned}
 &100 + (.75)100 + (.75)^2 100 + (.75)^3 100 + \dots \\
 &= 100 \left[1 + (.75) + (.75)^2 + (.75)^3 + \dots \right] \\
 &= 100 \cdot \frac{1}{1 - (.75)} = \left(400\right) \text{ (million \$)}
 \end{aligned}$$