Math 16C
Kouba
Newton's Method

RECALL : Newton's Method is used to create a sequence of estimates for the solution r of the equation $f(x) = 0$. Start with an initial guess x_1 and then use

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{for } n = 1, 2, 3, 4, \cdots .$$

EXAMPLE : Estimate the value of the solution r to the equation $\ln x = 4 - x$.

We will solve

$f(x) = \ln x - (4 - x) = \ln x - 4 + x = 0$;

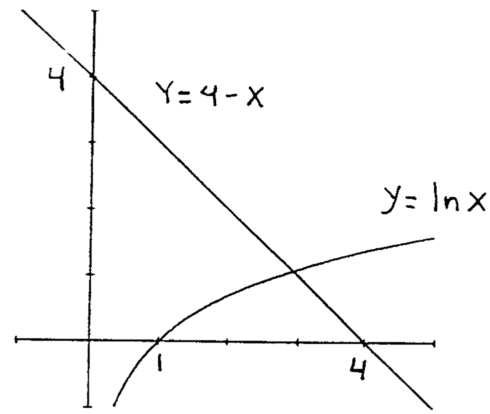
$f'(x) = \frac{1}{x} + 1 = \frac{1+x}{x}$, so Newton's Method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\ln(x_n) - 4 + x_n}{\frac{1+x_n}{x_n}}$$

$$= \frac{x_n(1+x_n)}{1+x_n} - \frac{(\ln(x_n) - 4 + x_n)\, x_n}{1+x_n}$$

$$= \frac{x_n + x_n^2 - x_n \ln(x_n) + 4x_n - x_n^2}{1+x_n} \quad \text{or} \quad x_{n+1} = \frac{5x_n - x_n \ln(x_n)}{1+x_n} .$$



Looking at the graphs of $y = \ln x$ and $y = 4 - x$, it appears that $x_1 = 2$ is a good first guess. Then

$$x_2 = \frac{5x_1 - x_1 \ln(x_1)}{1+x_1} = 2.8712352$$

$$x_3 = \frac{5x_2 - x_2 \ln(x_2)}{1+x_2} = 2.9261365$$

$$x_4 = \frac{5x_3 - x_3 \ln(x_3)}{1+x_3} = 2.9262711$$

so the solution r is approximately $r = 2.926$.