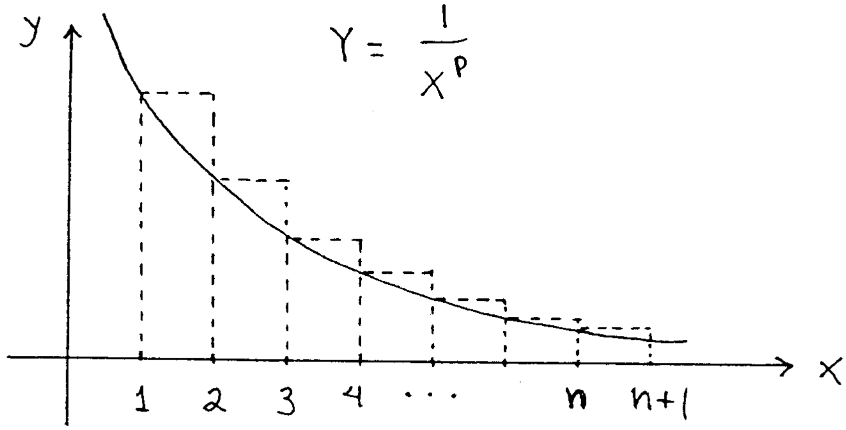
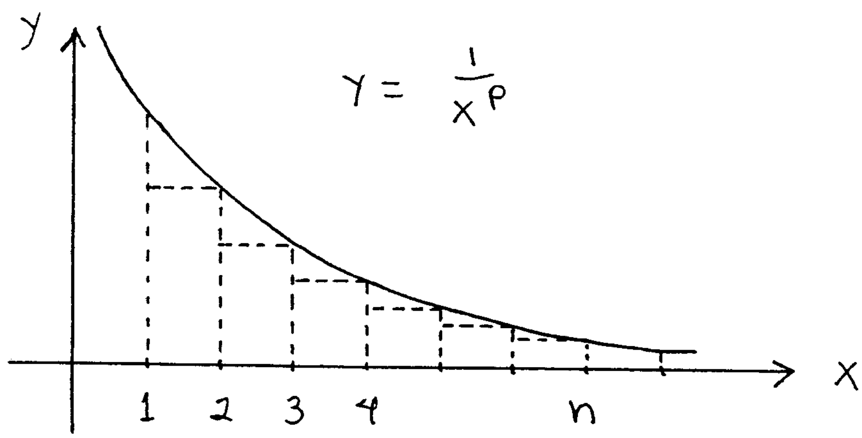
Math 16C
Kouba
p- series Handout



(A) $\displaystyle\int_1^{n+1} \frac{1}{x^p}\,dx < \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p}$ and



$\displaystyle\frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots \frac{1}{n^p} < \int_1^{n} \frac{1}{x^p}\,dx$ so that

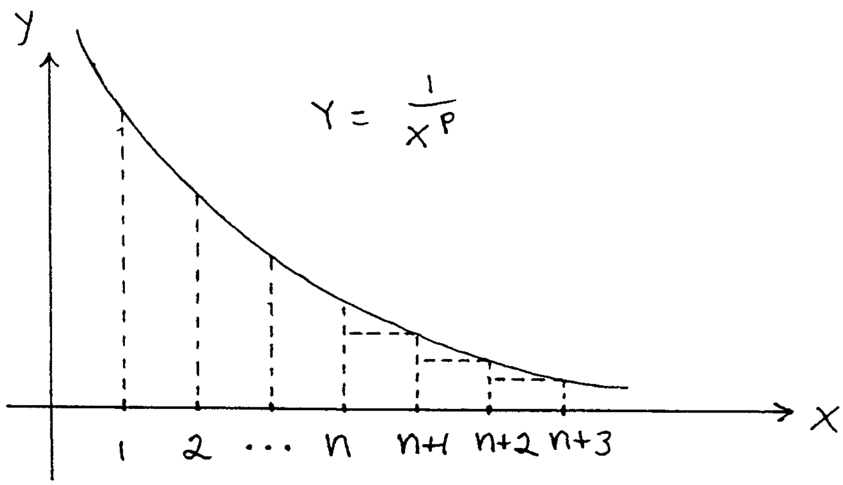
(B) $\displaystyle\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} < 1 + \int_1^{n} \frac{1}{x^p}\,dx$.

Letting $n \to \infty$ in (A) and (B) results in

$$\int_1^\infty \frac{1}{x^p}\,dx < \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots < 1 + \int_1^\infty \frac{1}{x^p}\,dx.$$

The results of the p-series test follow.

<u>Estimating Convergent p-Series</u>:



(*)
$$\frac{1}{(n+1)^p} + \frac{1}{(n+2)^p} + \frac{1}{(n+3)^p} + \cdots < \int_n^\infty \frac{1}{x^p}\,dx = \frac{n^{1-p}}{p-1}.$$