HOW DO WE ESTIMATE THE VALUE OF π ?
(An Application of Taylor Series)

A calculator will show that

$$\pi = 3.141592654 .$$

From where does this decimal number come ? The following list of Taylor series tells the story.

First, $$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \cdots ,$$

then $$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots ,$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots ,$$ and

$$\arctan x = x - x^3/3 + x^5/5 - x^7/7 + x^9/9 - x^{11}/11 + \cdots .$$

Letting $x = 1$, we get

$$\pi/4 = \arctan 1 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + \cdots$$

and

$$\pi = 4 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 + 4/13 - 4/15 + \cdots .$$

The more terms that are added, the more accurate the estimate for π. For example, the sum of the first 200,001 terms will guarantee accuracy of π to four decimal places.