Math 16C
Kouba
Taylor Polynomials

Ex: $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots$ so:

$P_0(x) = 1$

$P_2(x) = 1 - x^2$

$P_4(x) = 1 - x^2 - \frac{x^4}{2!}$

$P_6(x) = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$

$\vdots$

	$x = 0.5$	absolute error $\lvert f(x) - P_n(x) \rvert$
e^{-x^2}	0.778800783	
$P_0(x)$	1.000000000	0.221199217
$P_2(x)$	0.750000000	0.028800783
$P_4(x)$	0.78125000	0.002449217
$P_6(x)$	0.778645833	0.000154950
$\vdots$		

Ex: $\frac{x}{1-x} = x + x^2 + x^3 + x^4 + \cdots$ so:

$P_0(x) = 0$

$P_1(x) = x$

$P_2(x) = x + x^2$

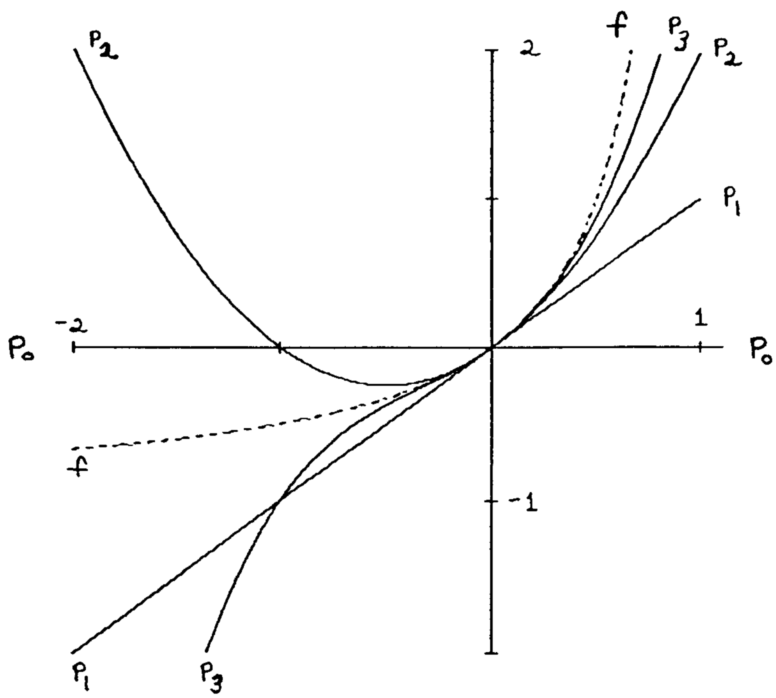
$P_3(x) = x + x^2 + x^3$

$P_4(x) = x + x^2 + x^3 + x^4$

$\vdots$

	$x = -0.2$	$\lvert f(x) - P_n(x) \rvert$	$x = 0.3$	$\lvert f(x) - P_n(x) \rvert$
$\frac{x}{1-x}$	-0.166666666		0.428571428	
$P_0(x)$	0.000000000	0.166666666	0.000000000	0.428571428
$P_1(x)$	-0.200000000	0.033333333	0.300000000	0.128571428
$P_2(x)$	-0.160000000	0.006666666	0.390000000	0.038571428
$P_3(x)$	-0.168000000	0.001333333	0.417000000	0.011571428
$P_4(x)$	-0.166400000	0.000266666	0.425100000	0.003471428
$\vdots$				

Kouba
Math 16C
Taylor Polynomials-- Patterns of Convergence



The graphs above suggest that , as n gets larger, the sequence of Taylor polynomials

$P_0(x) = 0$
$P_1(x) = x$
$P_2(x) = x + x^2$
$P_3(x) = x + x^2 + x^3$
$P_4(x) = x + x^2 + x^3 + x^4$
$\vdots$

become better and better approximations of the function

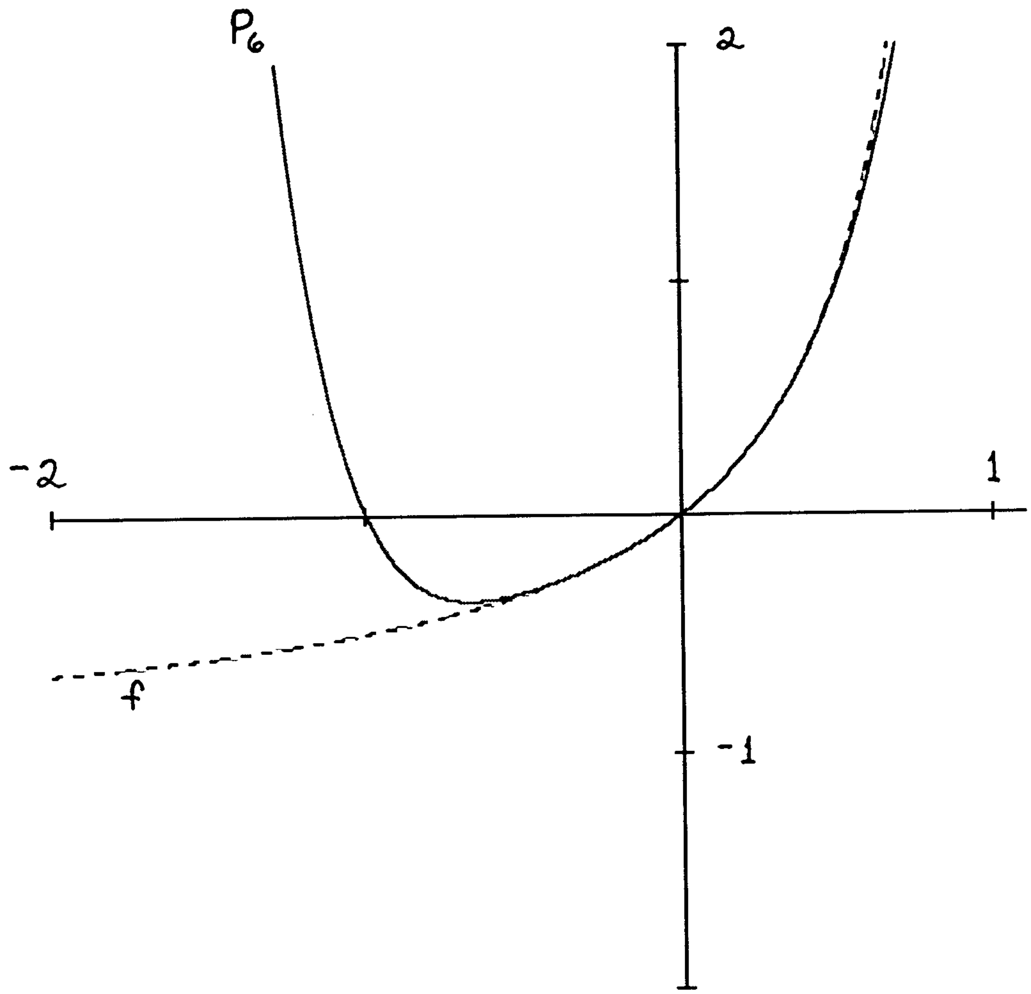
$$f(x) = \frac{x}{1 - x}$$

for x-values near x = 0.

More precisely, it can be stated that if $P_n(x)$ is the Taylor polynomial for f(x) of degree n centered at c, then

$$\left| f(x) - P_n(x) \right| \le \left| \frac{f^{(n+1)}(z)\,(x-c)^{n+1}}{(n+1)!} \right|$$

where z is some number between x and c .



The graphs of

$$f(x) = \frac{x}{1 - x}$$

and its 6th-degree Taylor polynomial

$$P_6(x) = x + x^2 + x^3 + x^4 + x^5 + x^6$$

are shown above. Note that the two graphs are virtually identical for x-values between -1/2 and 1/2.