Math 17A
Kouba
Beverton-Holt Growth Curve

Recall: For discrete exponential growth/decay we have

$$N_t = N_0 \cdot R^t \quad \text{for } t = 0, 1, 2, 3, \ldots$$

and for some constant R. Note also that

$$\begin{cases} N_t \text{ increases if } R > 1 \\ N_t \text{ decreases if } 0 < R < 1 \\ N_t \text{ is constant if } R = 1 \end{cases}.$$

In addition, we have

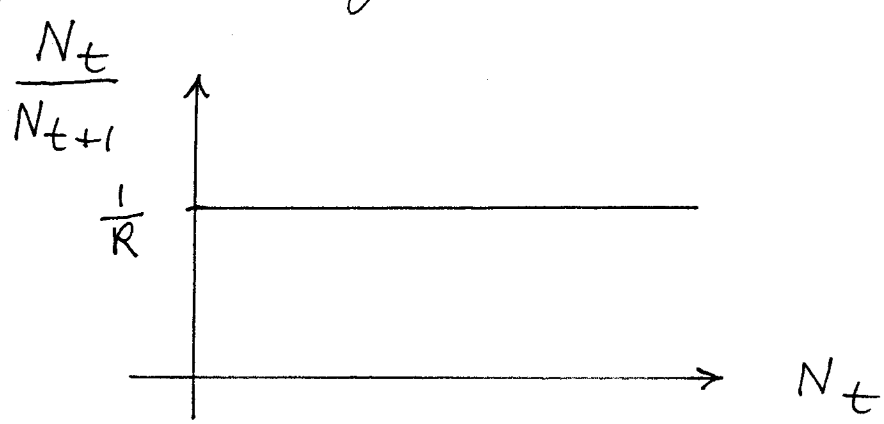
$$N_{t+1} = N_0 R^{t+1} = N_0 \cdot R \cdot R^t = R(N_0 R^t) = R \cdot N_t$$

so that $N_{t+1} = R N_t$ and

$$\boxed{\frac{N_t}{N_{t+1}} = \frac{1}{R}} \quad \text{for } t = 0, 1, 2, 3, \ldots .$$

We call $\frac{N_t}{N_{t+1}}$ the parent-offspring ratio. Since $\frac{1}{R}$ is a constant, we say the growth is density independent, i.e., the growth rate does <u>not</u> depend on the size of N_t at time t. In many cases, this is unrealistic due to limitations imposed by space, habitat, food, etc. It is more realistic to assume that the growth rate <u>decreases</u> as N_t increases.

It is useful to plot $\frac{N_t}{N_{t+1}}$ vs. N_t: For discrete exponential growth/decay we have



<u>Beverton-Holt Growth Curve</u>

This discrete model assumes that the graph of $\frac{N_t}{N_{t+1}}$ vs. N_t

is an increasing linear function, which implies :

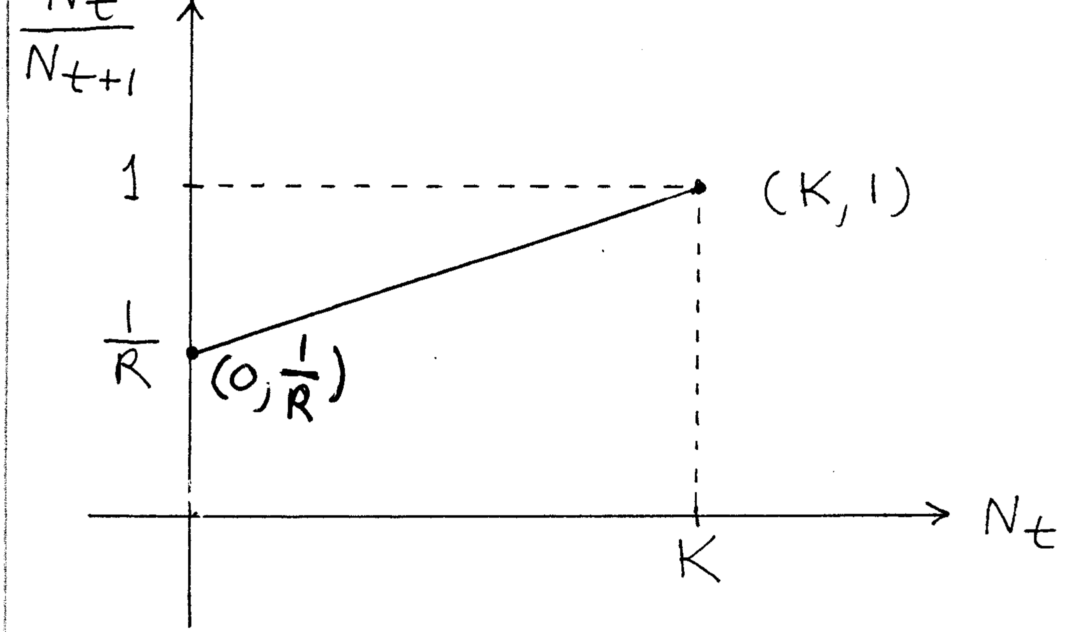
1.) $R > 1$ but decreases over time.

2.) The growth rate of N_t decreases over time.

3.) The growth of N_t is density dependent.

<u>Note</u>: If $\frac{N_t}{N_{t+1}} = 1$, then $N_{t+1} = N_t$,

which means there are no new members and N_t has reached its carrying capacity, K, i.e., $\boxed{\lim_{t \to \infty} N_t = K}$. The following graph is consistent with this information:



The equation of this line is :

$$Y = mX + b \longrightarrow$$

$$\frac{N_t}{N_{t+1}} = \frac{1 - 1/R}{K - 0} \cdot N_t + \frac{1}{R} \longrightarrow$$

$$\frac{R N_t}{N_{t+1}} = \frac{R-1}{K} \cdot N_t + 1 \longrightarrow$$

$$\boxed{N_{t+1} = \frac{R \cdot N_t}{1 + \frac{R-1}{K} \cdot N_t} \quad \text{for } t = 0, 1, 2, \ldots}$$

This is the Beverton-Holt Growth Recursion.