

Inflection Point - A Biological Interpretation

Ex: Assume that the number of elk in an isolated herd in Yellowstone National Park at time t (years) is given by

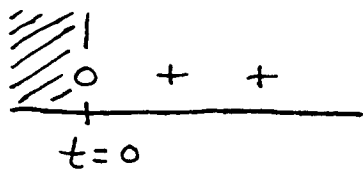
$$N(t) = 400 - \frac{4500}{t^2 + 30} \quad \text{for } t \geq 0.$$

Note that $N(0) = 250$ so initially there are 250 elk in the herd. Since

$$\lim_{t \rightarrow +\infty} N(t) = 400,$$

we would expect that after a "long period of time" the number of elk in the herd would stabilize at 400. Differentiation gives

$$N'(t) = 4500 (t^2 + 30)^{-2} (2t) = \frac{900t}{(t^2 + 30)^2},$$

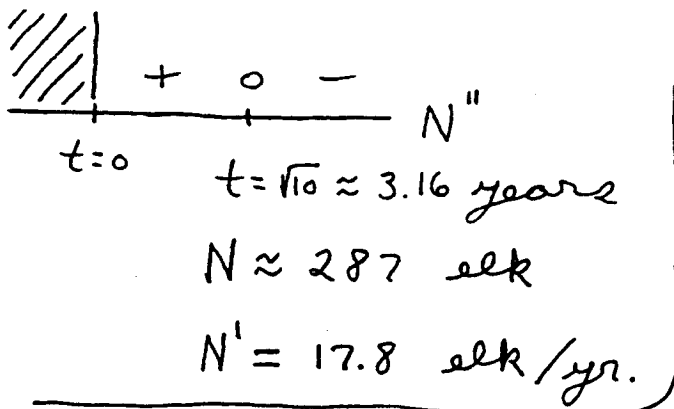

 N' ; since the units of $N'(t)$ are elk per year, the

following examples show the rate at which the herd is increasing at specific times:

$$\begin{aligned}
 N'(1) &= 9.4 \text{ elk per year,} \\
 N'(5) &= 14.9 \text{ elk per year,} \\
 N'(10) &= 5.3 \text{ elk per year.}
 \end{aligned}$$

Differentiating again,

$$N''(t) = \frac{9000 [30 - 3t^2]}{(t^2 + 30)^2}.$$



What is the biological significance of this inflection point?

It represents the time at which

the size of the elk herd is increasing most rapidly. The graph of $N(t)$ follows.

