

Math 17A
Kouba
Continuity

Def: Function $Y=f(x)$ is continuous at $x=a$ if

- i.) $f(a)$ exists
- ii.) $\lim_{x \rightarrow a} f(x)$ exists (finite),
- and iii.) $\lim_{x \rightarrow a} f(x) = f(a)$.

Fact: Sums, differences, products, quotients (denominator $\neq 0$), and compositions of continuous functions are continuous.

Fact: Every polynomial is continuous for all x -values.

Ex: Let $f(x) = 7x^5 - x^4 + 2x^3 - x + 20$; f is continuous for all values of x since it is a polynomial.

Ex: Let $f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3}$; since $Y = x^2 - 5x + 6$ (polynomial)

and $Y = 2x^2 + x - 3$ (polynomial) are continuous for all x -values, it follows that $f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3}$ is

continuous for all x -values except where

$Y = 2x^2 + x - 3 = (2x+3)(x-1) = 0$, that is, except at $x=1$ and $x = -3/2$.

Ex: Let $f(x) = (3 + \sin x)^{50}$; since $g(x) = 3 + \sin x$

(well known) and $h(x) = x^{50}$ (polynomial) are continuous

for all values of x , it follows that their composition

$f(x) = h(g(x)) = (3 + \sin x)^{50}$ is continuous for all values of x .