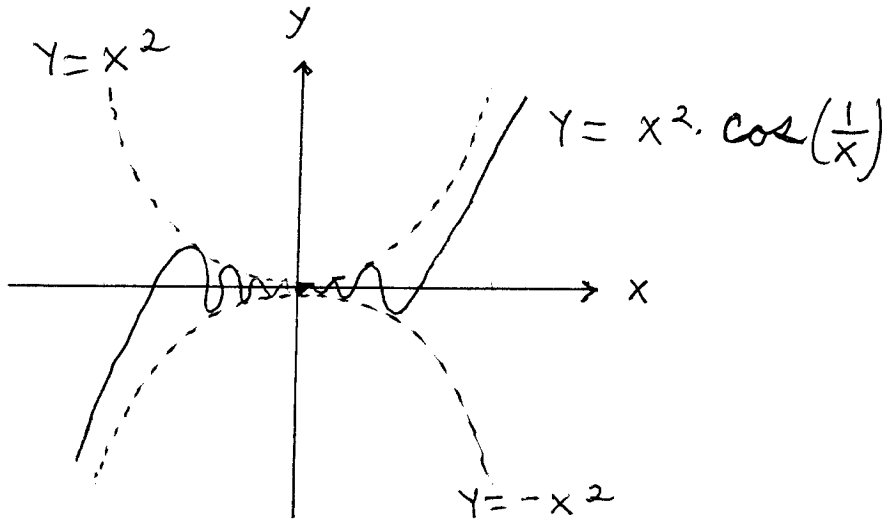
Section 3.4

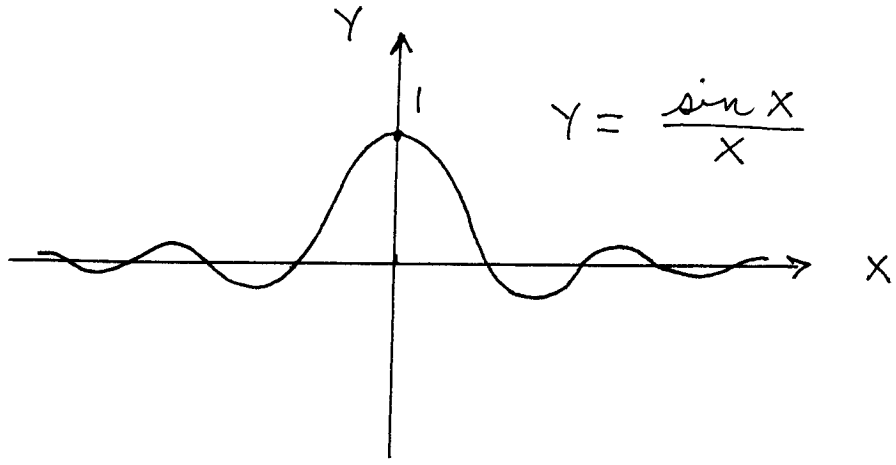
1.) a.)



b.) c.) $-1 \le \cos\left(\frac{1}{x}\right) \le +1 \rightarrow -x^2 \le x^2 \cos\left(\frac{1}{x}\right) \le x^2$;

$\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2$ so by Squeeze Principle $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$

4.) a.)



c.) $-1 \le \sin x \le +1 \rightarrow$ (if $x > 0$) $\rightarrow$

$\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$; $\lim_{x \to \infty} \frac{-1}{x} = 0 = \lim_{x \to \infty} \frac{1}{x}$

so by Squeeze Principle

$\lim_{x \to \infty} \frac{\sin x}{x} = 0$.

5.) $\lim_{x \to 0} \frac{\sin(2x)}{2x} = 1$

6.) $\lim_{x \to 0} \frac{\sin 2x}{3x} = \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \frac{2}{3}$

$= (1) \cdot \frac{2}{3} = \frac{2}{3}$

7.) $\lim_{x \to 0} \frac{\sin(5x)}{x} = \lim_{x \to 0} \frac{\sin(5x)}{5x} \cdot 5$

$= (1) \cdot 5 = 5$

8.) $\lim_{x \to 0} \frac{\sin x}{-x} = \lim_{x \to 0} -\left(\frac{\sin x}{x}\right) = -1$

9.) $\lim_{x \to 0} \frac{\sin(\pi x)}{x} = \lim_{x \to 0} \frac{\sin(\pi x)}{\pi x} \cdot \pi$

$= (1) \cdot \pi = \pi$

10.) $\lim_{x \to 0} \frac{\sin\left(-\frac{\pi}{2}x\right)}{2x} = \lim_{x \to 0} \frac{\sin\left(-\frac{\pi}{2}x\right)}{-\frac{\pi}{2}x} \cdot \frac{-\frac{\pi}{2}}{2}$

$= (1) \cdot \frac{-\pi}{2} \cdot \frac{1}{2} = \frac{-\pi}{4}$

11.) $\lim_{x \to 0} \frac{\sin \pi x}{\sqrt{x}} = \lim_{x \to 0} \frac{\sin \pi x}{\pi \cdot \sqrt{x}} \cdot \pi \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$= \lim_{x \to 0} \frac{\sin \pi x}{\pi x} \cdot \pi\sqrt{x} = (1) \cdot \pi(0) = 0$

12.) $\lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \sin x$

$= (1) \cdot \sin 0 = (1)(0) = 0$

13.) $\lim_{x\to 0} \frac{\sin x \cos x}{x(1-x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\cos x}{1-x}$

$= (1) \cdot \frac{\cos 0}{1-0} = (1) \cdot \frac{1}{1} = 1$

14.) $\lim_{x\to 0} \frac{1-\cos^2 x}{x^2} = \lim_{x\to 0} \frac{\sin^2 x}{x^2}$

$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} = (1)\cdot(1) = 1$

15.) $\lim_{x\to 0} \frac{1-\cos x}{2x} = \lim_{x\to 0} \frac{1}{2} \cdot \frac{1-\cos x}{x}$

$= \frac{1}{2}(0) = 0$

16.) $\lim_{x\to 0} \frac{1-\cos 2x}{3x} = \lim_{x\to 0} \frac{1-\cos 2x}{2x} \cdot \frac{2}{3}$

$= (0) \cdot \frac{2}{3} = 0$

19.) $\lim_{x\to 0} \frac{\sin x \cdot (1-\cos x)}{x^2} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1-\cos x}{x}$

$= (1)\cdot(0) = 0$

20.) $\lim_{x\to 0} \frac{\csc x - \cot x}{x \csc x} = \lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x \cdot \frac{1}{\sin x}} \cdot \frac{\sin x}{\sin x}$

$= \lim_{x\to 0} \frac{1-\cos x}{x} = 0$