

Section 3.5

2.) Let $f(x) = x^3 - 2x + 3$ on $[-3, -1]$;

f is continuous on $[-3, -1]$ since it is a polynomial;

then $x^3 - 2x + 3 = 0$

so let $m = 0$ and

we have $f(-1) = 4$

and $f(-3) = -18$ with

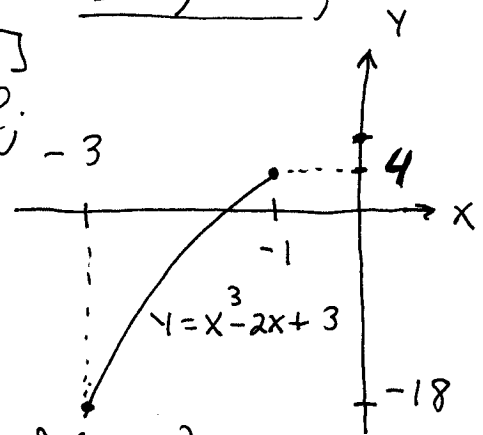
$m = 0$ between $f(-1)$ and $f(-3)$.

It follows from the IMVT that

there is a $\# c$, $-3 \leq c \leq -1$, so

that $f(c) = m$, i.e., $c^3 - 2c + 3 = 0$

and the equation is solvable.



4.) Let $f(x) = \sin x - x$ on $[-1, 1]$;

$y = \sin x$ is a well-known continuous function

and $y = x$ is a

polynomial (continuous),

so f is continuous,

because it is the

difference of continuous functions;

then $\sin x - x = 0$ so let $m = 0$ and

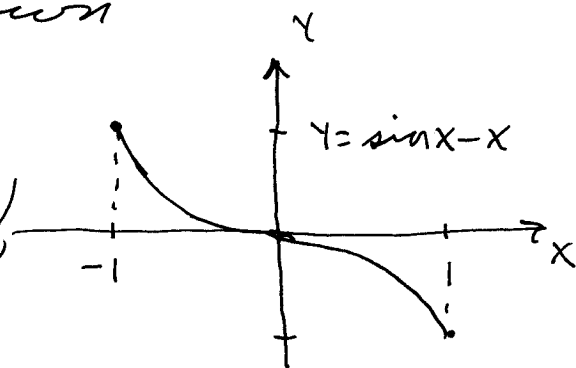
we have $f(1) = \sin 1 - 1 \approx -0.16$ and

$f(-1) = \sin(-1) + 1 \approx 0.16$ with $m = 0$

between $f(1)$ and $f(-1)$. It follows

from the IMVT that there is a

$\# c$, $-1 \leq c \leq 1$, so that $f(c) = m$,



i.e., $\sin c - c = 0$ and the equation is solvable.

5.) $e^{-x} - x = 0$ so choose $f(x) = e^{-x} - x$ and let $m=0$; $y = e^{-x}$ (well-known) and $y = x$ (polynomial) are continuous for all values of x , so $f(x) = e^{-x} - x$ is continuous for all values of x since it is the difference of continuous functions; note that $f(0) = e^0 - 0 = 1$ and $f(1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0$ and $m=0$ is between $f(0)$ and $f(1)$, so choose the interval $[0, 1]$; by the IMVT there is a $\# c$, $0 \leq c \leq 1$, so that $f(c) = m$; i.e., $e^{-c} - c = 0$ and the equation is solvable.

7.) (Calculator Problem) $f(x) = e^{-x} - x$:

<u>a</u>	<u>b</u>	<u>$\frac{1}{2}(a+b)$</u>	<u>f(a)</u>	<u>f(b)</u>	<u>$f(\frac{1}{2}(a+b))$</u>
0	1	0.5	+ 1	-0.632	+ 0.107
0.5	1	0.75	+ 0.107	-0.632	- 0.278
0.5	0.75	0.625	+ 0.107	-0.278	- 0.090
0.5	0.625	0.563	+ 0.107	-0.090	+ 0.007
0.563	0.625	0.594	+ 0.007	-0.090	- 0.041
0.563	0.594	0.578	+ 0.007	-0.041	- 0.017
0.563	0.578	0.571	+ 0.007	-0.017	- 0.006
0.563	0.571	0.567	+ 0.007	-0.006	- 0.0002
0.563	0.567	0.565	+ 0.007	-0.0002	+ 0.003
0.565	0.567	0.566	+ 0.003	-0.0002	+ 0.002
0.566	0.567	→			

solution ≈ 0.56 or 0.57

9.) a.) (Calculator Problem)

If $3x^3 - 4x^2 - x + 3 = 0$, then let
 $f(x) = 3x^3 - 4x^2 - x + 3$ and let $m = 0$;
 f is continuous (polynomial) for all
 x -values; note that $f(0) = 3$ and
 $f(-1) = -3$ and $m = 0$ is between $f(0)$
and $f(-1)$ so choose interval $[-1, 0]$;
then by IMVT it follows that
there is a $\# c$, $-1 \leq c \leq 0$, so that
 $f(c) = m$, i.e., $3c^3 - 4c^2 - c + 3 = 0$ and
the equation is solvable.

Estimate value of solution using
a spreadsheet: $f(x) = 3x^3 - 4x^2 - x + 3 = 0$

(Estimate solution to 2 decimal places.)

$$\text{Solve } f(x) = 3x^3 - 4x^2 - x + 3 = 0$$

a	b	(1/2)(a+b)	f(a)	f(b)	f((1/2)(a+b))
-1	0	-0.5	-3	3	2.125
-1	-0.5	-0.75	-3	2.125	0.234375
-1	-0.75	-0.875	-3	0.234375	-1.197265625
-0.875	-0.75	-0.8125	-1.197265625	0.234375	-0.4372558594
-0.8125	-0.75	-0.78125	-0.4372558594	0.234375	-0.0906677246
-0.78125	-0.75	-0.765625	-0.0906677246	0.234375	0.0745124817
-0.78125	-0.765625	-0.7734375	-0.0906677246	0.0745124817	-0.0074086189
-0.7734375	-0.765625	-0.76953125	-0.0074086189	0.0745124817	0.0337186456
-0.7734375	-0.76953125	-0.771484375	-0.0074086189	0.0337186456	0.0131967589
-0.7734375	-0.771484375	-0.7724609375	-0.0074086189	0.0131967589	0.0029045148
-0.7734375	-0.7724609375	-0.7729492188	-0.0074086189	0.0029045148	-0.0022494398

so solution ≈ -0.77

(Estimate solution to 2 decimal places.)

$$\text{Solve : } f(x) = \ln(x+1) + x - 1 = 0$$

a	b	(1/2)(a+b)	f(a)	f(b)	f((1/2)(a+b))
0	1	0.5	-1	0.6931471806	-0.0945348919
0.5	1	0.75	-0.0945348919	0.6931471806	0.3096157879
0.5	0.75	0.625	-0.0945348919	0.3096157879	0.1105078158
0.5	0.625	0.5625	-0.0945348919	0.1105078158	0.0087871026
0.5	0.5625	0.53125	-0.0945348919	0.0087871026	-0.0426656047
0.53125	0.5625	0.546875	-0.0426656047	0.0087871026	-0.0168882332
0.5625	0.546875	0.5546875	0.0087871026	-0.0168882332	-0.0040379392
0.5625	0.5546875	0.55859375	0.0087871026	-0.0040379392	0.0023777224
0.5546875	0.55859375	0.556640625	-0.0040379392	0.0023777224	-0.0008293212
0.556640625	0.55859375	0.5576171875	-0.0008293212	0.0023777224	0.0007743971

so solution ≈ 0.55 or 0.56