Section 4.2

1.) $f(x) = 4x^3 - 7x + 1 \xrightarrow{D} f'(x) = 12x^2 - 7$

4.) $f(x) = -3x^4 + 6x^2 - 2 \xrightarrow{D} f'(x) = -12x^3 + 12x$

6.) $f(x) = -1 + 3x^2 - 2x^4 \xrightarrow{D} f'(x) = 6x - 8x^3$

10.) $h(t) = \frac{1}{2}t^2 - 3t + 2 \xrightarrow{D} h'(t) = t - 3$

16.) $f(x) = \frac{1}{2}x^2e^3 - x^4 \xrightarrow{D} f'(x) = x \cdot e^3 - 4x^3$

18.) $f(x) = \frac{1}{e}x + e^2x + e \xrightarrow{D} f'(x) = \frac{1}{e} + e^2$

19.) $f(x) = 20x^3 - 4x^6 + 9x^8 \xrightarrow{D}$
$f'(x) = 60x^2 - 24x^5 + 72x^7$

20.) $f(x) = \frac{1}{15}x^3 - \frac{1}{20}x^4 + \frac{2}{15} \xrightarrow{D}$
$f'(x) = \frac{1}{5}x^2 - \frac{1}{5}x^3$

25) $f(x) = ax^2 - 2a \xrightarrow{D} f'(x) = 2ax$

30.) $f(x) = \frac{1}{rs^2}(r+x) - rsx + (r+s)x - rs \xrightarrow{D}$

$f'(x) = \frac{1}{rs^2}(1) - rs + (r+s)$

32.) $f(N) = \frac{1}{K+b}(bN^2 + N) \xrightarrow{D}$

$$f'(N) = \frac{1}{K+b}(2bN+1)$$

37.) $g(N) = N\left(1 - \frac{N}{K}\right) = N - \frac{1}{K}N^2 \xrightarrow{D}$

$$g'(N) = 1 - \frac{2}{K}N$$

44.) $Y = -2x^3 - 3x + 1 \xrightarrow{D} Y' = -6x^2 - 3$
and pt. $X = 1 \rightarrow Y = -2 - 3 + 1 = -4$ and
slope of tangent line is
$m = f'(1) = -6 - 3 = -9$ so line is
$Y - (-4) = -9(X - 1) \rightarrow Y + 4 = -9X + 9 \rightarrow$
$Y = -9X + 5$

50.) $Y = 1 - 3X^2 \xrightarrow{D} Y' = -6X$ and pt.
$X = -2 \rightarrow Y = 1 - 3(4) = -11$ and $f'(-2) = 12$
so slope of $\perp$ line is $m = -1/12$ so
$\perp$ line is $Y - (-11) = \frac{-1}{12}(X - (-2)) \rightarrow$
$Y + 11 = \frac{-1}{12}X - \frac{1}{6} \rightarrow Y = \frac{-1}{12}X - \frac{67}{6}$

60.) $f(X) = aX^2 - 3aX$ and $X = 2 \rightarrow$
$y = f(2) = 4a - 6a = -2a$; $\xrightarrow{D}$
$f'(X) = 2aX - 3a \rightarrow$ <u>tangent</u> SLOPE
is $f'(2) = 4a - 3a = a$ so NORMAL ($\perp$)
SLOPE is $m = -1/a$; equation
of normal line is
$y - (-2a) = \frac{-1}{a}(X - 2) \rightarrow Y + 2a = \frac{-1}{a}(X - 2)$

64.) $f(x) = 2 - x^2 \xrightarrow{D}$

$f'(x) = -2x = 0 \rightarrow \quad x = 0, y = 2$

has horizontal tangent

67.) $f(x) = 3x^3 - x^2 \xrightarrow{D}$

$f'(x) = 9x^2 - 2x = x(9x - 2) = 0$

$\rightarrow x = 0, y = 0 \quad$ OR $\quad x = \frac{2}{9}, y = \frac{-4}{243}$

have horizontal tangents

69.) $f(x) = \frac{1}{2}x^4 - \frac{7}{3}x^3 - 2x^2 \xrightarrow{D}$

$f'(x) = \frac{1}{2} \cdot 4x^3 - \frac{7}{3} \cdot 3x^2 - 4x$

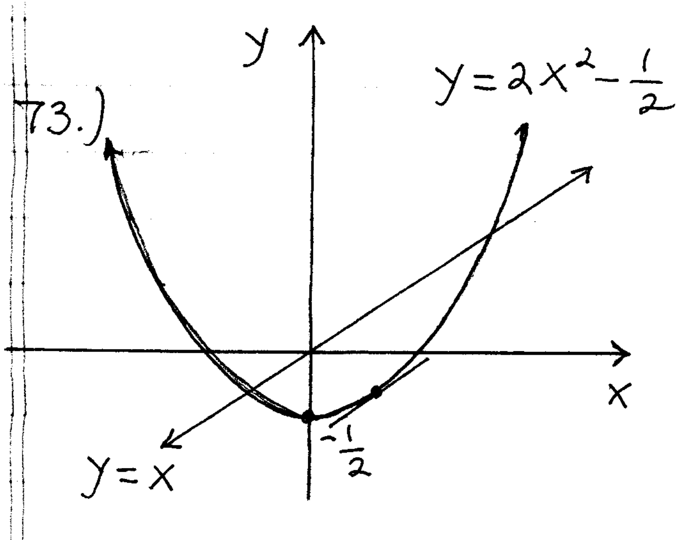
$= 2x^3 - 7x^2 - 4x = x(2x^2 - 7x - 4)$

$= x(2x + 1)(x - 4) = 0 \rightarrow$

$x = 0, y = 0 \quad$ OR $\quad x = 4, y = \frac{-160}{3} \quad$ OR $\quad x = \frac{-1}{2}, y = \frac{-17}{96}$

have horizontal tangents

73.)



$y = x = 1 \cdot x + 0$

has slope $= 1$;

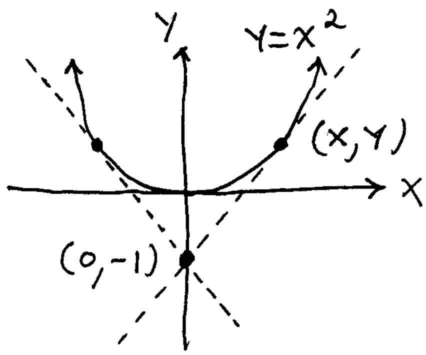
$y = 2x^2 - \frac{1}{2} \xrightarrow{D}$

$y' = 4x$ (slope of tangent line at x)

so $4X = 1 \rightarrow \quad X = \frac{1}{4}, \; Y = 2\left(\frac{1}{4}\right)^2 - \frac{1}{2} \rightarrow$
$Y = \frac{1}{8} - \frac{4}{8} = -\frac{3}{8}$

75.) Line $3X - Y = 2 \rightarrow Y = 3X - 2$ has slope $m = 3$; $\quad Y = X^3 + 2X + 2 \xrightarrow{D}$
$Y' = 3X^2 + 2 = 3 \longrightarrow 3X^2 = 1 \rightarrow X^2 = 1/3 \rightarrow$
$X = \frac{1}{\sqrt{3}}, \; Y = \frac{7+6\sqrt{3}}{3\sqrt{3}}$ and $X = \frac{-1}{\sqrt{3}}, \; Y = \frac{6\sqrt{3}-7}{3\sqrt{3}}$

78.)



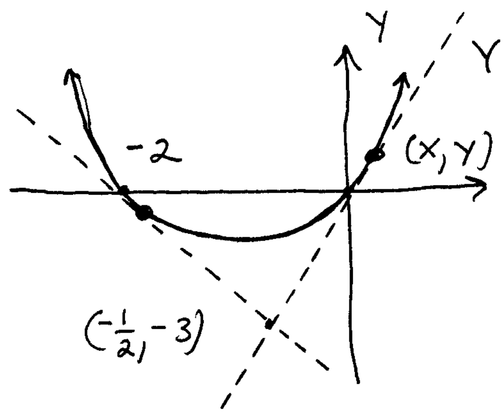
$Y = X^2 \xrightarrow{D} Y' = 2X$;
SLOPE of tangent at pt. (X, Y) is

i.) $m = 2X$

or ii.) $m = \frac{Y-(-1)}{X-0} = \frac{X^2+1}{X}$;

set slopes equal getting
$2X = \frac{X^2+1}{X} \longrightarrow 2X^2 = X^2 + 1 \longrightarrow$
$X^2 = 1 \longrightarrow \quad X = 1, Y = 1$ or $X = -1, Y = 1$
have tangent lines passing through the point $(0, -1)$.

80.)



$Y = X^2 + 2X \xrightarrow{D} Y' = 2X + 2$;

SLOPE of tangent at pt. (X, Y) is

i.) $m = 2X + 2$

OR ii.) $m = \frac{Y - (-3)}{X - (-1/2)} = \frac{X^2 + 2X + 3}{X + 1/2}$;

set slopes equal getting

$$2X + 2 = \frac{X^2 + 2X + 3}{X + 1/2} \rightarrow (2X + 2)(X + \tfrac{1}{2}) = X^2 + 2X + 3$$

$$\rightarrow 2X^2 + X + 2X + 1 = X^2 + 2X + 3$$

$$\rightarrow X^2 + X - 2 = 0 \rightarrow (X - 1)(X + 2) = 0 \rightarrow$$

$X = 1, Y = 3$ or $X = -2, Y = 0$ have tangent lines passing through the point $(-1/2, -3)$.