Section 4.4

2.) $f(x) = (4x+5)^3 \xrightarrow{D} f'(x) = 3(4x+5)^2 \cdot 4$

5.) $f(x) = (x^2+3)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(x^2+3)^{-1/2} \cdot 2x$

9.) $f(x) = \dfrac{1}{(x^3-2)^4} = (x^3-2)^{-4} \xrightarrow{D}$

$f'(x) = -4(x^3-2)^{-5} \cdot 3x^2$

12.) $f(x) = \dfrac{(1-2x^2)^3}{(3-x^2)^2} \xrightarrow{D}$

$$f'(x) = \frac{(3-x^2)^2 \cdot 3(1-2x^2)^2 \cdot (-4x) - (1-2x^2)^3 \cdot 2(3-x^2) \cdot (-2x)}{(3-x^2)^4}$$

16.) $g(t) = (t^2 + (t+1)^{1/2})^{1/2} \xrightarrow{D}$

$g'(t) = \frac{1}{2}(t^2+(t+1)^{1/2})^{-1/2} \cdot \left\{2t + \frac{1}{2}(t+1)^{-1/2} \cdot (1)\right\}$

17.) $g(t) = \left(\dfrac{t}{t-3}\right)^3 \xrightarrow{D}$

$g'(t) = 3\left(\dfrac{t}{t-3}\right)^2 \cdot \dfrac{(t-3)(1) - t(1)}{(t-3)^2}$

19.) $f(r) = (r^2-r)^3 \cdot (r+3r^3)^{-4} \xrightarrow{D}$

$f'(r) = (r^2-r)^3 \cdot -4(r+3r^3)^{-5} \cdot (1+9r^2)$

$\quad + 3(r^2-r)^2 \cdot (2r-1) \cdot (r+3r^3)^{-4}$

24.) $f(x) = (2-4x^2)^{1/4} \xrightarrow{D}$

$f'(x) = \frac{1}{4}(2-4x^2)^{-3/4} \cdot (-8x)$

27.) $h(t) = \left(3t + \frac{3}{t}\right)^{2/5} \xrightarrow{D}$

$h'(t) = \frac{2}{5}\left(3t + \frac{3}{t}\right)^{-3/5} \cdot \left\{ 3 + \frac{t \cdot (0) - 3 \cdot (1)}{t^2} \right\}$

34.) a.) $f'(x) = 2x+1 \longrightarrow$

$D\, f(x^2) = f'(x^2) \cdot D(x^2)$

$= (2(x^2)+1) \cdot 2x$, let $x = -1 \longrightarrow$

$D\, f(x^2) = (2(-1)^2+1)(-2) = -6$

35.) b.) $f'(x) = \frac{1}{x} \longrightarrow$

$D\, f(\sqrt{x-1}) = f'(\sqrt{x-1}) \cdot D(\sqrt{x-1})$

$= \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2}(x-1)^{-1/2} \cdot (1) = \frac{1}{2\sqrt{x-1}\sqrt{x-1}} = \frac{1}{2(x-1)}$

37.) $D\left(\frac{f(x)}{g(x)} + 1\right)^2 = 2\left(\frac{f(x)}{g(x)} + 1\right) \cdot \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

38.) $D\, f\left(\frac{1}{g(x)}\right) = f'\left(\frac{1}{g(x)}\right) \cdot \frac{g(x) \cdot (0) - (1) \cdot g'(x)}{(g(x))^2}$

$= f'\left(\frac{1}{g(x)}\right) \cdot \frac{-g'(x)}{(g(x))^2}$

43.) $Y = (1 + (3x^2 - 1)^3)^2 \xrightarrow{D}$

$Y' = 2(1 + (3x^2-1)^3) \cdot \{0 + 3(3x^2-1)^2 \cdot (6x)\}$

47.) $x^2 + Y^2 = 4 \xrightarrow{D} 2x + 2YY' = 0 \longrightarrow$

$$2YY' = -2X \rightarrow Y' = \frac{-2X}{2Y} \rightarrow Y' = -\frac{X}{Y}$$

48.) $Y = X^2 + 3XY \xrightarrow{D}$

$Y' = 2X + 3XY' + (3)Y \rightarrow Y' - 3XY' = 2X + 3Y \rightarrow$

$(1-3X)Y' = 2X+3Y \rightarrow Y' = \dfrac{2X+3Y}{1-3X}$

50.) $XY - Y^3 = 1 \xrightarrow{D} XY' + (1)Y - 3Y^2 \cdot Y' = 0 \rightarrow$

$XY' - 3Y^2Y' = -Y \rightarrow (X - 3Y^2)Y' = -Y \rightarrow$

$Y' = \dfrac{-Y}{X-3Y^2}$

51.) $x^{1/2}y^{1/2} = x^2 + 1 \xrightarrow{D}$

$x^{1/2} \cdot \frac{1}{2}y^{-1/2} \cdot y' + \frac{1}{2}x^{-1/2} \cdot y^{1/2} = 2x \rightarrow$

$\frac{1}{2}\sqrt{x} \cdot \frac{1}{\sqrt{y}}\, y' = 2x - \frac{1}{2}\frac{1}{\sqrt{x}}\sqrt{y} \rightarrow$

$$y' = \frac{2x - \frac{1}{2}\sqrt{y/x}}{\frac{1}{2}\sqrt{\frac{x}{y}}}$$

52.) $\dfrac{1}{2xy} - y^3 = 4 \rightarrow$ (mult. by $2xy$) $\rightarrow$

$1 - 2xy^4 = 8xy \xrightarrow{D}$

$0 - (2x \cdot 4y^3 \cdot y' + 2 \cdot y^4) = 8x \cdot y' + 8 \cdot y \rightarrow$

$-8xy^3y' - 2y^4 = 8xy' + 8y \rightarrow$ (mult by $\frac{1}{2}$)

$-4xy^3y' - y^4 = 4xy' + 4y \rightarrow$

$-y^4 - 4y = 4xy^3y' + 4xy' \rightarrow$

$y'(4xy^3 + 4x) = -y^4 - 4y \rightarrow$

$$y' = \frac{-y^4 - 4y}{4xy^3 + 4x}$$

53.) $\frac{x}{y} = \frac{y}{x} \rightarrow x^2 = y^2 \xrightarrow{D} \cancel{2}x = \cancel{2}yy' \rightarrow$

$y' = x/y$

54.) $\frac{x}{xy+1} = 2xy \rightarrow x = 2xy(xy+1) \rightarrow$

$x = 2x^2 \cdot y^2 + 2x \cdot y \xrightarrow{D}$

$1 = 2x^2 \cdot 2yy' + 4x \cdot y^2 + 2x \cdot y' + 2y \rightarrow$

$1 - 4xy^2 - 2y = (4x^2y + 2x)y' \rightarrow$

$$y' = \frac{1 - 4xy^2 - 2y}{4x^2y + 2x}$$

55.) $x^2 + y^2 = 25 \xrightarrow{D} 2x + 2yy' = 0 \rightarrow$

$2yy' = -2x \rightarrow y' = \frac{-2x}{2y} \rightarrow y' = \frac{-x}{y}$;

at ~~pt.~~ $(4, -3) \rightarrow y' = \frac{-(4)}{-3} = 4/3$;

a.) $m = y' = \frac{4}{3}$ so tangent line is

$y - (-3) = \frac{4}{3}(x - 4) \rightarrow y + 3 = \frac{4}{3}x - \frac{16}{3} \rightarrow$

$y = \frac{4}{3}x - \frac{25}{3}$

60.) a.) $Y^2 = 10x^4 - x^2 \xrightarrow{D}$

$2YY' = 40x^3 - 2x \rightarrow Y' = \dfrac{40x^3 - 2x}{2Y} \rightarrow$

$Y' = \dfrac{\cancel{2}(20x^3 - x)}{\cancel{2}Y} \rightarrow Y' = \dfrac{20x^3 - x}{Y}$;

at pt. $(1,3) \rightarrow Y' = \dfrac{20(1)^3 - 1}{3} = \dfrac{19}{3}$

61.) $x^2 + y^2 = 1$, $\dfrac{dx}{dt} = 2$, $x = 1/2 \rightarrow$

$\left(\frac{1}{2}\right)^2 + y^2 = 1 \rightarrow y^2 = \frac{3}{4} \rightarrow y = \dfrac{\pm\sqrt{3}}{2}$

$(y > 0)$ so $y = \dfrac{+\sqrt{3}}{2}$; then $\xrightarrow{D}$

$\cancel{2}x \cdot \dfrac{dx}{dt} + \cancel{2}y \dfrac{dy}{dt} = 0 \rightarrow \left(\frac{1}{2}\right)(2) + \left(\frac{\sqrt{3}}{2}\right) \cdot \dfrac{dy}{dt} = 0$

$\rightarrow \dfrac{\sqrt{3}}{2} \cdot \dfrac{dy}{dt} = -1 \rightarrow \dfrac{dy}{dt} = \dfrac{-2}{\sqrt{3}}$.

63.) $x^2 y = 1$, $\dfrac{dx}{dt} = 3$, $x = 2 \rightarrow 4y = 1 \rightarrow$

$y = 1/4$; then $\xrightarrow{D} x^2 \cdot \dfrac{dy}{dt} + 2x \cdot \dfrac{dx}{dt} \cdot y = 0$

$\rightarrow (2)^2 \cdot \dfrac{dy}{dt} + 2(2)(3)\left(\frac{1}{4}\right) = 0 \rightarrow \dfrac{dy}{dt} = \dfrac{-3}{4}$

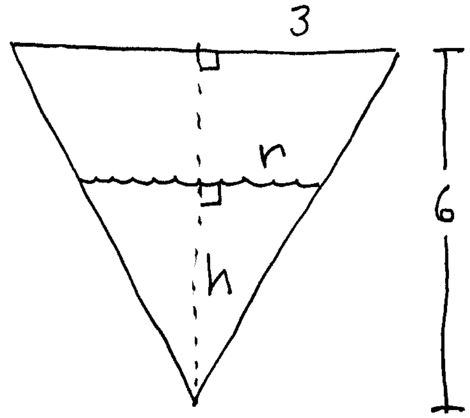
65.) $V = x^3 \xrightarrow{D} \dfrac{dV}{dt} = 3x^2 \cdot \dfrac{dx}{dt}$

69.) $V = 25\pi h$ and $\frac{dV}{dt} = \frac{250\,\ell.}{min.} \cdot \frac{1\,m.^3}{1000\,\ell.} \rightarrow$

$\frac{dV}{dt} = \frac{1}{4} \frac{m.^3}{min.}$; $\xrightarrow{D}$ $\frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt} \rightarrow$

$\frac{1}{4} = 25\pi \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{1}{100\pi}$ m./min.

70.)



By similar Δ's

$\frac{r}{h} = \frac{3}{6} \rightarrow \boxed{r = \frac{1}{2}h}$;

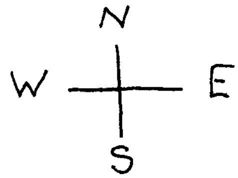
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h \rightarrow$

$\boxed{V = \frac{1}{12}\pi h^3}$; assume

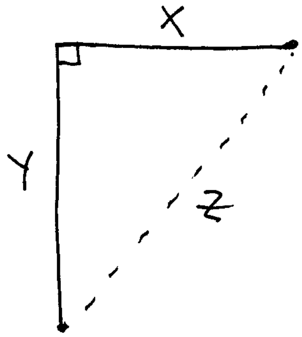
$\frac{dV}{dt} = 5$ ft.^{3}/min., find $\frac{dh}{dt}$ when $h = 2$ ft.:

$V = \frac{1}{12}\pi h^3 \xrightarrow{D} \frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt} \rightarrow$

$5 = \frac{\pi}{\cancel{4}}(\cancel{2})^{\cancel{2}} \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{5}{\pi}$ ft./min.



71.)



Assume $\underline{\frac{dX}{dt} = 15 \text{ mph}}$, $\underline{\frac{dY}{dt} = 18 \text{ mph}}$; find $\frac{dz}{dt}$ when $t = \frac{1}{3}$ hr., $t = \frac{2}{3}$ hr. :

$X^2 + Y^2 = z^2 \xrightarrow{D}$

$$2X \cdot \frac{dX}{dt} + 2Y \cdot \frac{dY}{dt} = 2z \frac{dz}{dt} \rightarrow X \cdot \frac{dX}{dt} + Y \cdot \frac{dY}{dt} = z \cdot \frac{dz}{dt}$$

$$\rightarrow \boxed{\frac{dz}{dt} = \frac{1}{z}\left(X \cdot \frac{dX}{dt} + Y \cdot \frac{dY}{dt}\right)} \quad (*) \quad ;$$

a.) If $t = \frac{1}{3}$ hr. $\rightarrow X = (15)\left(\frac{1}{3}\right) = 5$ mi., $Y = (18)\left(\frac{1}{3}\right) = 6$ mi., and $(5)^2 + (6)^2 = z^2 \rightarrow$ $z^2 = 61 \rightarrow z = \sqrt{61} \approx 7.8$ mi. $\rightarrow$ (plug in $(*)$)

$$\rightarrow \frac{dz}{dt} = \frac{1}{\sqrt{61}}\left((5)\cdot(15) + (6)(18)\right) \approx 23.4 \text{ mph}$$

b.) If $t = \frac{2}{3}$ hr. $\rightarrow X = (15)\left(\frac{2}{3}\right) = 10$ mi., $Y = (18)\left(\frac{2}{3}\right) = 12$ mi., and $(10)^2 + (12)^2 = z^2 \rightarrow$ $z^2 = 244 \rightarrow z = \sqrt{244} \approx 15.6$ mi. $\rightarrow$ (plug in $(*)$)

$$\rightarrow \frac{dz}{dt} = \frac{1}{\sqrt{244}}\left((10)(15) + (12)(18)\right) \approx 23.4 \text{ mph}$$

74.) $f(x) = (2x^2+4)^3 \xrightarrow{D} f'(x) = 3(2x^2+4)^2 \cdot 4x$

$= 12x(2x^2+4)^2 \xrightarrow{D}$

$f''(x) = 12x \cdot 2(2x^2+4) \cdot 4x + (12)(2x^2+4)^2$

$= 12(2x^2+4)[8x^2 + (2x^2+4)]$

$= 12(2x^2+4)[10x^2+4]$

80.) $f(x) = \dfrac{2x}{x^2+1} \xrightarrow{D} f'(x) = \dfrac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$

$= \dfrac{2-2x^2}{(x^2+1)^2} \xrightarrow{D} f''(x) = \dfrac{(x^2+1)^2(-4x) - (2-2x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \dfrac{-4x(x^2+1)\left[(x^2+1)+(2-2x^2)\right]}{(x^2+1)^4} = \dfrac{-4x(3-x^2)}{(x^2+1)^3}$

86.) a.) $s(t) = t^2 - 3t \xrightarrow{D}$

vel. $s'(t) = 2t - 3 \rightarrow s'(1) = 2-3 = -1$; $\xrightarrow{D}$

acc. $s''(t) = 2 \rightarrow s''(1) = 2$

b.) $s(t) = \sqrt{t^2+1} \xrightarrow{D}$

vel. $s'(t) = \frac{1}{2}(t^2+1)^{-1/2}\cdot(2t) = \dfrac{t}{\sqrt{t^2+1}} \rightarrow$

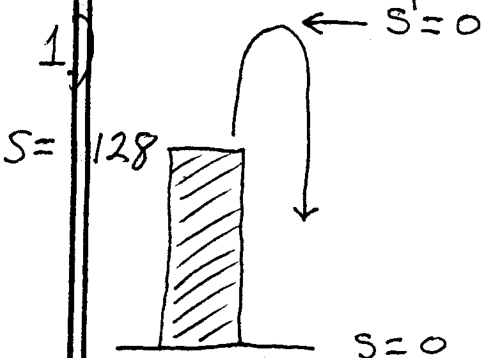
$s'(1) = 1/\sqrt{2}$; $\xrightarrow{D}$

acc. $s''(t) = \dfrac{\sqrt{t^2+1}\cdot(1) - t\cdot\frac{1}{2}(t^2+1)^{-1/2}\cdot 2t}{(\sqrt{t^2+1})^2}$

$= \dfrac{\dfrac{\sqrt{t^2+1}}{1} - \dfrac{t^2}{\sqrt{t^2+1}}}{\dfrac{t^2+1}{1}} = \dfrac{\cancel{t^2}+1-\cancel{t^2}}{(t^2+1)^{1/2}} \cdot \dfrac{1}{(t^2+1)^1} \rightarrow$

$s''(t) = \dfrac{1}{(t^2+1)^{3/2}} \rightarrow s''(1) = \dfrac{1}{2^{3/2}}$

Gravity Problems

1.) 

$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 112t + 128}$

$\xrightarrow{D} \boxed{s'(t) = -32t + 112}$

a.) highest point : $s'(t) = 0$

$\rightarrow -32t + 112 = 0 \rightarrow t = \frac{112}{32} = \underline{3.5 \text{ sec}} \rightarrow$

$s(3.5) = -16(3.5)^2 + 112(3.5) + 128 = 324 \text{ ft.}$

b.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 112t + 128 = 0 \rightarrow -16(t^2 - 7t - 8) = 0 \rightarrow$

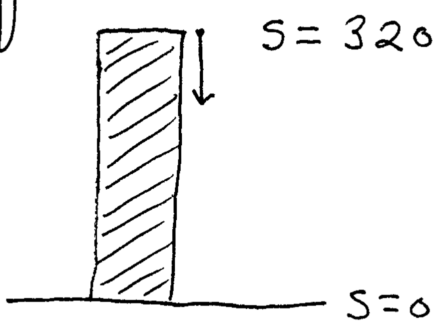
$-16(t-8)(t+1) = 0 \rightarrow t = -1$ (NO) or

$t = 8$ sec.

c.) i.) $s'(3) = -32(3) + 112 = 16$ ft./sec

ii.) $s'(4) = -32(4) + 112 = -16$ ft./sec.

iii.) $s'(8) = -32(8) + 112 = -144$ ft./sec.

2.) 

$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 - 16t + 320} \rightarrow$

$\boxed{s'(t) = -32t - 16}$

a.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 - 16t + 320 = 0 \rightarrow -16(t^2 + t - 20) = 0 \rightarrow$

$-16(t-4)(t+5) = 0 \rightarrow t = -5$ (NO) or

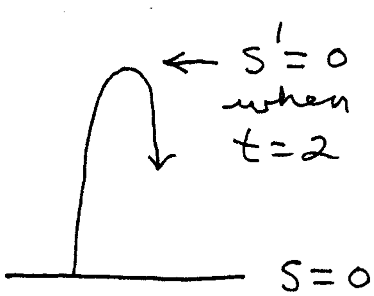
$t = 4$ sec.

b.) i.) $s'(1) = -32(1) - 16 = -48$ ft./sec.

ii.) $s'(2) = -32(2) - 16 = -80$ ft./sec.

iii.) $s'(4) = -32(4) - 16 = -144$ ft./sec.

3.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + v_0 t} \rightarrow$

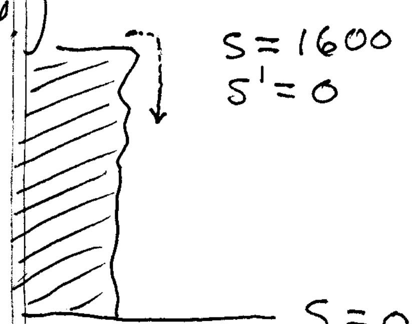
$\boxed{s'(t) = -32t + v_0}$

and $s'(2) = 0 \rightarrow -32(2) + v_0 = 0$

$\rightarrow$ b.) $v_0 = 64$ ft./sec.

a.) $s(2) = -16(2)^2 + 64(2) = 64$ ft.

4.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 1600} \rightarrow$

$\boxed{s'(t) = -32t}$

a.) hit ground : $s(t) = 0 \rightarrow$

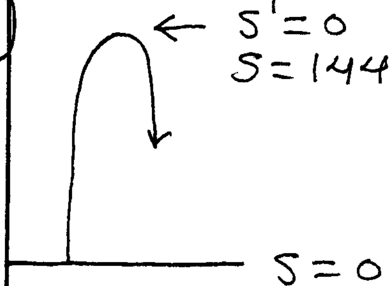
$-16t^2 + 1600 = 0 \rightarrow 16t^2 = 1600 \rightarrow t^2 = 100 \rightarrow$

$t = 10$ sec.

b.) i.) $s'(5) = -32(5) = -160$ ft./sec.

ii.) $s'(10) = -32(10) = -320$ ft./sec ;

$$-\frac{320 \text{ ft.}}{\text{sec}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx -218.2 \text{ mph}$$

5.) 

$s(t) = -16t^2 + V_0 t + S_0 \rightarrow$

$\boxed{s(t) = -16t^2 + V_0 t} \rightarrow$

$\boxed{s'(t) = -32t} + V_0;$

Let $t = T$ be time required to reach highest point. Then

$$\left.\begin{aligned} s'(T) &= -32T + V_0 = 0 \\ s(T) &= -16T^2 + V_0 T = 144 \end{aligned}\right\} \rightarrow$$

$\boxed{V_0 = 32T} \xrightarrow{(SUB)} -16T^2 + (32T)T = 144 \rightarrow$

$16T^2 = 144 \rightarrow T^2 = 9 \rightarrow T = 3$ sec.

a.) $T = 3$ sec

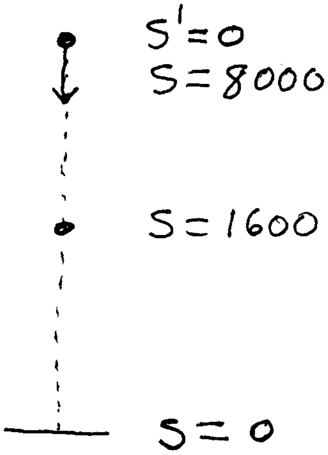
c.) $V_0 = 32T = 32(3) = 96$ ft./sec.

b.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 96t = 0 \rightarrow 16t(6-t) = 0 \rightarrow$

$t = 0$ and $t = 6$ sec.

d.) You know.

6.) 

$s(t) = -16t^2 + V_0 t + S_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 8000} \rightarrow$

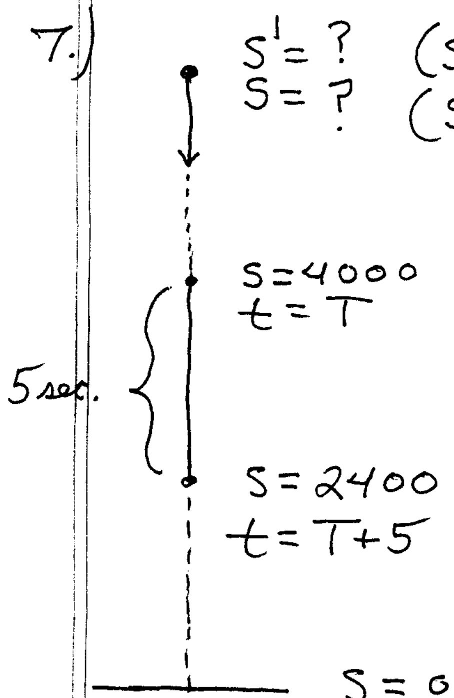
$\boxed{s'(t) = -32t}$

a.) $s(t) = 1600 \rightarrow -16t^2 + 8000 = 1600$

$\rightarrow 16t^2 = 6400 \rightarrow t^2 = 400 \rightarrow t = 20 \text{ sec.}$

b.) $s'(20) = -32(20) = -640$ ft./sec.

7.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$s'(t) = -32t + v_0$;

a.) $s'(10) = -400 \rightarrow$

$-32(10) + v_0 = -400 \rightarrow$

$v_0 = -80$ ft./sec.

b.) $\left.\begin{array}{l} s(T) = 4000 \\ s(T+5) = 2400 \end{array}\right\} \rightarrow$

$\left.\begin{array}{r} -16T^2 - 80T + s_0 = 4000 \\ -16(T+5)^2 - 80(T+5) + s_0 = 2400 \end{array}\right\} \rightarrow$

$\boxed{s_0 = 4000 + 80T + 16T^2} \rightarrow$ (SUB) $\rightarrow$

$-16(T^2 + 10T + 25) - 80T - 400$

$+ (4000 + 80T + 16T^2) = 2400 \rightarrow$

$\cancel{-16T^2} - 160T - 400 - \cancel{80T} - 400$

$+ 4000 + \cancel{80T} + \cancel{16T^2} = 2400 \rightarrow$

$160T = 800 \rightarrow T = 5$ sec. $\rightarrow$

$s_0 = 4000 + 80(5) + 16(5)^2 \rightarrow$

$s_0 = 4800$ ft.

c.) $s(t) = -16t^2 - 80t + 4800$,

<u>strike ground</u>: $s(t) = 0 \rightarrow$

$-16t^2 - 80t + 4800 = 0 \rightarrow$

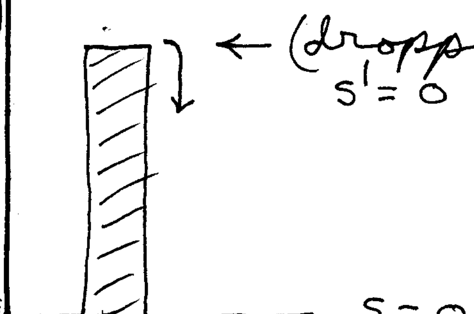
$-16(t^2 + 5t - 300) = 0 \rightarrow$

$-16(t-15)(t+20) = 0 \rightarrow$

$t = 15$ sec. $\qquad t \cancel{=} -20$

d.) Snapple Peach Ice Tea

8.)



Let H be height of building. Then

$s(t) = -16t^2 + (0)t + H \rightarrow$

$\boxed{s(t) = -16t^2 + H} \xrightarrow{D}$

$\boxed{s'(t) = -32t}$;

a.) Given $s(5) = 0$ ft. $\rightarrow -16(5)^2 + H = 0$

$\rightarrow \boxed{H = 400 \text{ ft}}$

b.) $s'(1) = -32(1) = -32$ ft./sec. ;

$s'(3) = -32(3) = -96$ ft./sec.

c.) $s'(5) = -32(5) = -160$ ft./sec.

$= \dfrac{-160 \text{ ft.}}{\text{sec.}} \times \dfrac{1 \text{ mi.}}{5280 \text{ ft.}} \times \dfrac{3600 \text{ sec.}}{1 \text{ hr.}} \approx 109.1$ mph