Section 4.6

1.) $f(x) = e^{3x} \xrightarrow{D} f'(x) = e^{3x} \cdot 3$

4.) $f(x) = 3e^{2-5x} \xrightarrow{D} f'(x) = 3 \cdot e^{2-5x} \cdot (-5)$

8.) $f(x) = e^{-3(x^3-1)^4} \xrightarrow{D}$
$f'(x) = e^{-3(x^3-1)^4} \cdot -12(x^3-1)^3 \cdot 3x^2$

9.) $f(x) = xe^x \xrightarrow{D} f'(x) = xe^x + e^x$

11.) $f(x) = x^2e^{-x} \xrightarrow{D} f'(x) = x^2 \cdot -e^{-x} + 2x \cdot e^{-x}$

13.) $f(x) = \dfrac{1+e^x}{1+x^2} \xrightarrow{D}$

$$f'(x) = \frac{(1+x^2)(e^x) - (1+e^x)(2x)}{(1+x^2)^2}$$

14.) $f(x) = \dfrac{x-e^{-x}}{1+xe^{-x}} \xrightarrow{D}$

$$f'(x) = \frac{(1+xe^{-x})\cdot(1- -e^{-x}) - (x-e^{-x})(x \cdot -e^{-x} + 1 \cdot e^{-x})}{(1+xe^{-x})^2}$$

17.) $f(x) = e^{\sin(3x)} \xrightarrow{D} f'(x) = e^{\sin(3x)} \cdot \cos(3x) \cdot 3$

20.) $f(x) = e^{\cos(1-2x^3)} \xrightarrow{D}$
$f'(x) = e^{\cos(1-2x^3)} \cdot -\sin(1-2x^3) \cdot (-6x^2)$

21.) $f(x) = \sin(e^x) \xrightarrow{D} f'(x) = \cos(e^x) \cdot e^x$

24.) $f(x) = \cos(3x - e^{x^2-1}) \xrightarrow{D}$

$f'(x) = -\sin(3x - e^{x^2-1}) \cdot \{3 - e^{x^2-1} \cdot 2x\}$

29.) $f(x) = e^{x \sin x} \xrightarrow{D}$

$f'(x) = e^{x \sin x} \cdot \{x \cdot \cos x + 1 \cdot \sin x\}$

33.) $f(x) = 2^x \xrightarrow{D} f'(x) = 2^x \ln 2$

37.) $f(x) = 5^{(2x-1)^{1/2}} \xrightarrow{D} f'(x) = 5^{(2x-1)^{1/2}} \cdot \frac{1}{2}(2x-1)^{-1/2} \cdot 2 \cdot \ln 5$

42.) $h(t) = 4^{2t^3 - t} \xrightarrow{D}$

$h'(t) = \underline{4^{2t^3-t}} \cdot (6t^2 - 1) \cdot \ln 4$

49.) $g(x) = 2^{2\cos x} \xrightarrow{D}$

$g'(x) = 2^{2\cos x} \cdot -2\sin x \cdot \ln 2$

53.) If $f(x) = e^{2x}$, then

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{e^{2(x+h)} - e^{2x}}{h}$; if $x = 0$ then

$\underline{\underline{f'(0)}} = \lim_{h \to 0} \frac{e^{2h} - e^0}{h} = \underline{\underline{\lim_{h \to 0} \frac{e^{2h} - 1}{h}}}$;

if $f(x) = e^{2x} \xrightarrow{D} f'(x) = e^{2x} \cdot 2 \longrightarrow$

$\underline{\underline{f'(0)}} = e^0 \cdot 2 = \underline{\underline{2}} \longrightarrow \lim_{h \to 0} \frac{e^{2h} - 1}{h} = 2$

54.) If $f(x) = e^{5x}$, then

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{e^{5(x+h)} - e^{5x}}{h}$; if $x = 0$ then

$\underline{\underline{f'(0)}} = \lim_{h \to 0} \frac{e^{5h} - e^0}{h} = \underline{\underline{\lim_{h \to 0} \frac{e^{5h} - 1}{h}}}$;

if $f(x) = e^{5x} \xrightarrow{D} f'(x) = e^{5x} \cdot 5 \longrightarrow$

$\underline{\underline{f'(0)}} = e^0 \cdot 5 = \underline{\underline{5}}$; then

$\lim_{h \to 0} \frac{e^{5h} - 1}{3h} = \lim_{h \to 0} \frac{1}{3} \cdot \frac{e^{5h} - 1}{h} = \frac{1}{3}(5) = \frac{5}{3}$

55.) If $f(x) = e^x$ then

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$; if $x = 0$ then

$\underline{\underline{f'(0)}} = \lim_{h \to 0} \frac{e^h - e^0}{h} = \underline{\underline{\lim_{h \to 0} \frac{e^h - 1}{h}}}$;

if $f(x) = e^x$, then $f'(x) = e^x \longrightarrow$

$\underline{\underline{f'(0)}} = e^0 = \underline{\underline{1}}$; then

$$\lim_{h\to 0} \frac{e^h - 1}{\sqrt{h}} = \lim_{h\to 0} \frac{\sqrt{h}}{\sqrt{h}} \cdot \frac{e^h - 1}{\sqrt{h}}$$

$$= \lim_{h\to 0} \sqrt{h} \cdot \frac{e^h - 1}{h} = \left(\lim_{h\to 0} \sqrt{h}\right)\left(\lim_{h\to 0} \frac{e^h - 1}{h}\right)$$

$$= \sqrt{0} \cdot (1) = 0$$

56.) If $f(x) = 2^x$, then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} \text{ ; if } x=0, \text{ then}$$

$$\underline{\underline{f'(0)}} = \lim_{h\to 0} \frac{2^h - 2^0}{h} = \underline{\underline{\lim_{h\to 0} \frac{2^h - 1}{h}}} \text{ ;}$$

if $f(x) = 2^x \xrightarrow{D} f'(x) = 2^x \ln 2 \longrightarrow$

$\underline{\underline{f'(0)}} = 2^0 \ln 2 = \underline{\underline{\ln 2}}$; then

$$\lim_{h\to 0} \frac{2^h - 1}{h} = \ln 2 .$$

59.) $N(t) = e^{2t}$

a.) $t = 0 \longrightarrow N = e^0 = 1$

b.) $\frac{dN}{dt} = e^{2t} \cdot 2 = 2e^{2t} = 2N$

60.) $N(t) = N_0 \cdot e^{rt}$

a.) $t=0 \rightarrow N = N_0 \cdot e^0 = N_0 \cdot (1) = N_0$

b.) $\frac{dN}{dt} = N_0 \cdot e^{rt} \cdot r = r(N_0 e^{rt}) = rN$

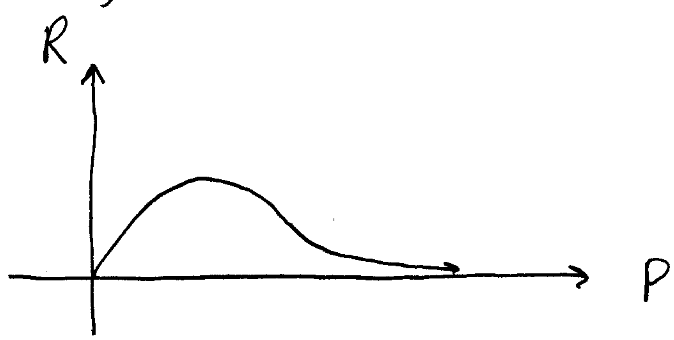
61.) $N(t) = N(0) \cdot 2^t \xrightarrow{D}$

$\frac{dN}{dt} = N(0) \cdot 2^t \cdot \ln 2 = \ln 2 \left(N_0 \cdot 2^t\right)$

$= \ln 2 \cdot N$, i.e., $\frac{dN}{dt} = \ln 2 \cdot N$

and growth rate $\frac{dN}{dt}$ is proportional to the population size N

64.) a.) $R(P) = 2P \cdot e^{-P}$



b.) $R'(P) = 2P \cdot e^{-P} \cdot (-1) + 2 \cdot e^{-P}$

$= 2(1-P)e^{-P}$

c.) $R'(P) = 0$ (horizontal tangent)

$\rightarrow 2(1-P)e^{-P} = 0 \rightarrow 1-P=0 \rightarrow P=1$

70.) $\frac{dw}{dt} = -2\,w(t)$ and $w(0) = 15 \rightarrow \boxed{W = 15e^{-2t}}$

a.) If $t = 2 \rightarrow W = 15e^{-2(2)} = 15e^{-4} \approx 0.275$

b.) If $W = 7.5 \longrightarrow 7.5 = 15e^{-2t} \rightarrow \frac{1}{2} = e^{-2t} \rightarrow$

$\ln(1/2) = \ln(e^{-2t}) = -2t \rightarrow t = \frac{-1}{2}\ln\left(\frac{1}{2}\right) \approx 0.347$

71.) $\frac{dw}{dt} = -3\,w(t)$ and $w(0) = 6 \longrightarrow$

$$\boxed{w = 6e^{-3t}}$$

a.) If $t = 4 \longrightarrow w = 6e^{-3(4)} = 6e^{-12} \approx 0.00004$

b.) If $w = 3$ $\left(\frac{1}{2} \text{ of } 6\right) \longrightarrow$

$3 = 6e^{-3t} \longrightarrow \frac{1}{2} = e^{-3t} \longrightarrow$

$\ln(1/2) = \ln(e^{-3t}) = -3t \rightarrow$

$t = \frac{-1}{3}\ln(1/2) \approx 0.231$

72.) assume $W = w_0 e^{kt}$ and

$t = 0,\ w = 10 \rightarrow 10 = w_0 \cdot e^0 = w_0 \cdot (1) = w_0$

$\rightarrow w = 10e^{kt}$; and $t = 1,\ w = 8$

$\rightarrow 8 = 10e^k \rightarrow 4/5 = e^k \longrightarrow$

$\ln(4/5) = \ln e^k = k \longrightarrow$

$$\boxed{w = 10e^{\ln(4/5)\cdot t}}$$

a.) $\frac{dw}{dt} = \ln(4/5) \cdot w$

b.) If $t = 5 \rightarrow w = 10e^{\ln(4/5)\cdot 5} \approx 3.277$

c.) If $w = 5$ $(\frac{1}{2} \text{ of } 10) \rightarrow 5 = 10e^{\ln(4/5)\cdot t}$

$\rightarrow \frac{1}{2} = e^{\ln(4/5)\cdot t} \rightarrow \ln(1/2) = \ln e^{\ln(4/5)\cdot t}$

$\rightarrow \ln(1/2) = \ln(4/5)\cdot t \rightarrow$

$t = \dfrac{\ln(1/2)}{\ln(4/5)} \approx 3.11$

73.) assume $w = w_0 e^{kt}$ and

$t=0,\ w=5 \rightarrow 5 = w_0 \cdot e^0 = w_0\cdot(1) = w_0 \rightarrow$

$w = 5e^{kt}$; $t=1,\ w=2 \rightarrow 2 = 5e^k \rightarrow$

$\frac{2}{5} = e^k \rightarrow \ln(2/5) = \ln e^k = k \rightarrow$

$$\boxed{w = 5e^{\ln(2/5)\cdot t}}$$

a.) $\dfrac{dw}{dt} = \ln(2/5)\cdot w$

b.) If $t = 3 \rightarrow w = 5e^{\ln(2/5)\cdot 3} = 0.32$

c.) If $w = 2.5$ $(\frac{1}{2} \text{ of } 5) \rightarrow 2.5 = 5e^{\ln(2/5)\cdot t}$

$\rightarrow \frac{1}{2} = e^{\ln(2/5)\cdot t} \rightarrow \ln(1/2) = \ln e^{\ln(2/5)\cdot t}$

$\rightarrow \ln(1/2) = \ln(2/5)\cdot t \rightarrow$

$t = \dfrac{\ln(1/2)}{\ln(2/5)} \approx 0.756$

I.) assume $N = N_0 e^{kt}$ and $t=0$ hrs.,
$N = 50$ g. $\rightarrow$ $N = 50e^{kt}$; and
$t = 4$ hrs., $N = 85$ g. $\rightarrow$ $85 = 50e^{4k} \rightarrow$
$\frac{85}{50} = e^{4k} \rightarrow \frac{17}{10} = e^{4k} \rightarrow$
$\ln(17/10) = \ln e^{4k} = 4k \rightarrow$
$k = \frac{1}{4}\ln(17/10) \rightarrow \boxed{N = 50e^{\frac{1}{4}\ln(17/10)\cdot t}}$;

a.) If $t = 6$ hrs. $\rightarrow$
$N = 50e^{\frac{1}{4}\ln(17/10)\cdot 6} \approx 110.8$ g.

b.) If $N = 200$ g. $\rightarrow$
$200 = 50e^{\frac{1}{4}\ln(17/10)\cdot t} \rightarrow$
$4 = e^{\frac{1}{4}\ln(17/10)\cdot t} \rightarrow$
$\ln 4 = \ln e^{\frac{1}{4}\ln(17/10)\cdot t} = \frac{1}{4}\ln(17/10)t \rightarrow$
$t = \frac{4\ln 4}{\ln(17/10)} \approx 10.45$ hrs.

II.) assume $N = N_0 e^{kt}$ and N_0 is the original amount ; and $t = 1600$ yrs.,
$N = \frac{1}{2}N_0 \rightarrow \frac{1}{2}\cancel{N_0} = \cancel{N_0}e^{1600k} \rightarrow$
$\ln(1/2) = \ln e^{1600k} = 1600k \rightarrow k = \frac{1}{1600}\ln(1/2)$;
$N = N_0 e^{\frac{1}{1600}\ln(1/2)t}$ and $N = 20\%$ of $N_0 = (0.2)N_0$

$\rightarrow (0.2)\cancel{N_0} = \cancel{N_0}\, e^{\frac{1}{1600}\ln(1/2)\cdot t}$

$\rightarrow \ln(0.2) = \ln e^{\frac{1}{1600}\ln(1/2)\cdot t} = \frac{1}{1600}\ln(1/2)\cdot t \rightarrow$

$$t = \frac{1600\ln(0.2)}{\ln(1/2)} \approx 3715 \text{ yrs.}$$

III.) assume $N = N_0 e^{kt}$ and N_0 is the original amount of carbon-14; and $t = 5730$ yrs. $\rightarrow N = \frac{1}{2}N_0 \rightarrow$

$\frac{1}{2}\cancel{N_0} = \cancel{N_0}\, e^{5730k} \rightarrow \ln(1/2) = \ln e^{5730k} = 5730k$

$\rightarrow k = \frac{1}{5730}\ln(1/2) \rightarrow N = N_0 e^{\frac{1}{5730}\ln(1/2)\cdot t}$;

if $t = 2006 - 1400 = 606$ yrs., then

$N = N_0 e^{\frac{1}{5730}\ln(1/2)\cdot 606} \approx 0.929\, N_0$ or

92.9% of the original amount