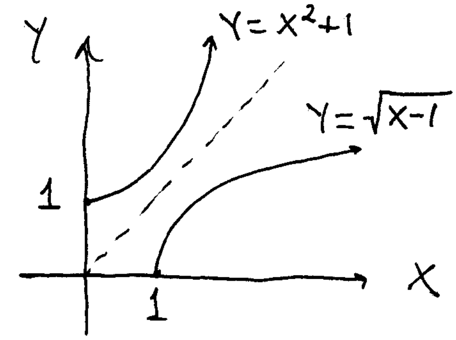
Section 4.7

2.) $f(x) = \sqrt{x-1} \rightarrow y = \sqrt{x-1} \rightarrow$ (switch) $\rightarrow$
$x = \sqrt{y-1} \rightarrow x^2 = y-1 \rightarrow y = x^2+1 \rightarrow$
$f^{-1}(x) = x^2+1$:



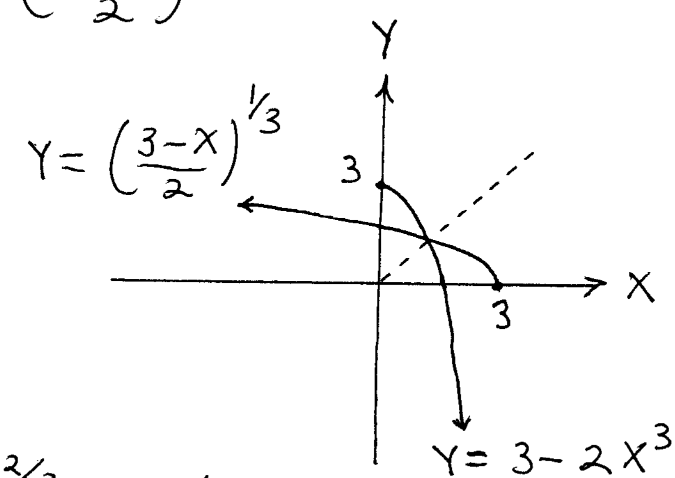
i.) $D f^{-1}(x) = D(x^2+1) = 2x$

ii.) $D f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))}$

$\left\{ f'(x) = \frac{1}{2}(x-1)^{-1/2} = \dfrac{1}{2\sqrt{x-1}} \right\}$

$= \dfrac{1}{\dfrac{1}{2\sqrt{(x^2+1)-1}}} = \dfrac{1}{\dfrac{1}{2\sqrt{x^2}}} = \dfrac{1}{\dfrac{1}{2|x|}} = 2|x| = 2x$ since $x \geq 0$

5.) $f(x) = 3-2x^3$ for $x \geq 0 \rightarrow y = 3-2x^3 \rightarrow$
(switch) $\rightarrow x = 3-2y^3 \rightarrow 2y^3 = 3-x \rightarrow$
$y^3 = \dfrac{3-x}{2} \rightarrow y = \left(\dfrac{3-x}{2}\right)^{1/3} \rightarrow$
$f^{-1}(x) = \left(\dfrac{3-x}{2}\right)^{1/3}$:



i.) $D f^{-1}(x) = D\left(\dfrac{3-x}{2}\right)^{1/3}$

$= \dfrac{1}{3}\left(\dfrac{3-x}{2}\right)^{-2/3} \cdot \left(\dfrac{-1}{2}\right)$

$= \dfrac{-1}{6} \cdot \dfrac{1}{\dfrac{(3-x)^{2/3}}{2^{2/3}}} = \dfrac{-2^{2/3}}{3 \cdot 2} \cdot \dfrac{1}{(3-x)^{2/3}}$

$= \dfrac{-1}{3 \cdot 2^{1/3}} \cdot \dfrac{1}{(3-x)^{2/3}}$

ii.) $D f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))}$ $\quad \{f'(x) = -6x^2\}$

$$= \frac{1}{-6\left[\left(\frac{3-x}{2}\right)^{1/3}\right]^2} = \frac{-1}{6 \cdot \frac{(3-x)^{2/3}}{2^{2/3}}} = \frac{-2^{2/3}}{3 \cdot 2 \cdot (3-x)^{2/3}}$$

$$= \frac{-1}{3 \cdot 2^{1/3}(3-x)^{2/3}}$$

7.) $f(x) = 2x^2 - 2$, $f(1) = 0 \rightarrow f^{-1}(0) = 1$, so

$D f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))}$ $\quad \{f'(x) = 4x\}$

$= \dfrac{1}{4 f^{-1}(x)}$ so $D f^{-1}(0) = \dfrac{1}{4 f^{-1}(0)} = \dfrac{1}{4 \cdot 1} = \dfrac{1}{4}$

11.) $f(x) = x + e^x$, $f'(x) = 1 + e^x$, $f(0) = 1 \longrightarrow$ $f^{-1}(1) = 0$, so

$D f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{1 + e^{f^{-1}(x)}}$, so

$D f^{-1}(1) = \dfrac{1}{1 + e^{f^{-1}(1)}} = \dfrac{1}{1 + e^0} = \dfrac{1}{1+1} = \dfrac{1}{2}$

15.) $f(x) = x^2 + \tan x$, $f'(x) = 2x + \sec^2 x$, $f(0) = 0 \rightarrow f^{-1}(0) = 0$, so

$D f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{2f^{-1}(x) + \sec^2(f^{-1}(x))} \longrightarrow$

$D f^{-1}(0) = \dfrac{1}{2f^{-1}(0) + \sec^2(f^{-1}(0))} = \dfrac{1}{2(0) + \sec^2 0}$

$$= \frac{1}{0+(1)^2} = 1$$

26.) $f(x) = \ln(4-3x) \xrightarrow{D} f'(x) = \frac{1}{4-3x} \cdot (-3)$

31.) $f(x) = (\ln x)^2 \xrightarrow{D} f'(x) = 2\ln x \cdot \frac{1}{x}$

34.) $f(x) = (\ln(1-x^2))^3 \xrightarrow{D}$

$f'(x) = 3(\ln(1-x^2))^2 \cdot \frac{1}{1-x^2} \cdot -2x$

36.) $f(x) = \ln(2x^2-x)^{1/2} \xrightarrow{D}$

$f'(x) = \frac{1}{(2x^2-x)^{1/2}} \cdot \frac{1}{2}(2x^2-x)^{-1/2} \cdot (4x-1)$

37.) $f(x) = \ln\left(\frac{x}{x+1}\right) \xrightarrow{D}$

$f'(x) = \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1) - x(1)}{(x+1)^2}$

45.) $f(x) = \ln(\tan(x^2)) \xrightarrow{D}$

$f'(x) = \frac{1}{\tan(x^2)} \cdot \sec^2(x^2) \cdot 2x$

46.) $g(s) = \ln(\sin^2(3s)) \xrightarrow{D}$

$g'(s) = \frac{1}{\sin^2(3s)} \cdot 2\sin(3s) \cdot \cos(3s) \cdot 3$

47.) $f(x) = x\ln x \xrightarrow{D} f'(x) = x \cdot \frac{1}{x} + (1)\ln x$

49.) $f(x) = \frac{\ln x}{x} \xrightarrow{D} f'(x) = \frac{x \cdot \frac{1}{x} - \ln x \cdot (1)}{x^2}$

51.) $h(t) = \sin(\ln(3t)) \xrightarrow{D}$

$h'(t) = \cos(\ln(3t)) \cdot \frac{1}{3t} \cdot 3$

58.) $f(x) = \log\, (\tan(x^2))^{1/3} \xrightarrow{D}$

$f'(x) = \frac{1}{(\tan(x^2))^{1/3}} \cdot \frac{1}{3}(\tan(x^2))^{-2/3} \cdot \sec^2(x^2) \cdot 2x \cdot \frac{1}{\ln 10}$

60.) $g(s) = \log_5 (3^s - 2) \xrightarrow{D}$

$= \frac{1}{3^s - 2} \cdot 3^s \cdot \ln 3 \cdot \frac{1}{\ln 5}$

63.) $f(x) = 2 \cdot x^x \rightarrow \ln f(x) = \ln(2 \cdot x^x) \rightarrow$

$\ln f(x) = \ln 2 + \ln x^x = \ln 2 + x \cdot \ln x \xrightarrow{D}$

$\frac{1}{f(x)} f'(x) = 0 + x \cdot \frac{1}{x} + (1) \ln x \rightarrow f'(x) = f(x)(1 + \ln x)$

$\rightarrow f'(x) = 2 \cdot x^x (1 + \ln x)$

67.) $f(x) = x^{\ln x} \longrightarrow \ln f(x) = \ln x^{\ln x} \longrightarrow$

$\ln f(x) = \ln x \cdot \ln x = (\ln x)^2 \xrightarrow{D}$

$\frac{1}{f(x)} \cdot f'(x) = 2(\ln x) \cdot \frac{1}{x} \longrightarrow$

$f'(x) = x^{\ln x}\left(\frac{2 \ln x}{x}\right) = 2x^{\ln x - 1} \cdot \ln x$

71.) $y = x^{x^x} \longrightarrow \ln y = \ln x^{x^x} = x^x \cdot \ln x \longrightarrow$

$\ln(\ln y) = \ln(x^x \cdot \ln x) \longrightarrow$

$\ln(\ln Y) = \ln x^x + \ln(\ln x) \longrightarrow$

$\ln(\ln Y) = x \cdot \ln x + \ln(\ln x) \xrightarrow{D}$

$\frac{1}{\ln Y} \cdot \frac{1}{Y} \cdot Y' = x \cdot \frac{1}{x} + (1)\ln x + \frac{1}{\ln x} \cdot \frac{1}{x} \longrightarrow$

$Y' = Y \ln Y \cdot \left[1 + \ln x + \frac{1}{x \ln x}\right]$

$= x^{x^x} \ln x^{x^x} \cdot \left[1 + \ln x + \frac{1}{x \ln x}\right]$

$= x^{x^x} \cdot x^x \cdot \ln x \cdot \left[1 + \ln x + \frac{1}{x \ln x}\right]$

72.) $Y = (x^x)^x = x^{x^2} \longrightarrow \ln Y = \ln x^{x^2} = x^2 \ln x \xrightarrow{D}$

$\frac{1}{Y} Y' = x^2 \cdot \frac{1}{x} + 2x \cdot \ln x \longrightarrow$

$Y' = x^{x^2} \cdot [x + 2x \ln x]$

73.) $Y = x^{\cos x} \longrightarrow \ln Y = \ln x^{\cos x} = \cos x \cdot \ln x$

$\xrightarrow{D} \frac{1}{Y} Y' = \cos x \cdot \frac{1}{x} + {}^{-}\sin x \cdot \ln x$

$\longrightarrow Y' = x^{\cos x} \cdot \left[\frac{\cos x}{x} - \sin x \cdot \ln x\right]$

76.) $Y = \frac{e^{x-1} \cdot \sin^2 x}{(x^2+5)^{2x}} \longrightarrow \ln Y = \ln\left(\frac{e^{x-1} \cdot \sin^2 x}{(x^2+5)^{2x}}\right)$

$= \ln(e^{x-1} \cdot \sin^2 x) - \ln(x^2+5)^{2x}$

$= \ln e^{x-1} + \ln(\sin x)^2 - 2x \cdot \ln(x^2+5)$

$= (x-1) + 2\ln(\sin x) - 2x \cdot \ln(x^2+5) \xrightarrow{D}$

$$\frac{1}{Y}Y' = 1 + 2 \cdot \frac{1}{\sin x} \cdot \cos x$$

$$- \left[2x \cdot \frac{1}{x^2+5} \cdot 2x + 2 \cdot \ln(x^2+5) \right] \rightarrow$$

$$Y' = \frac{e^{x-1} \sin^2 x}{(x^2+5)^{2x}} \cdot \left\{ 1 + 2\cot x - \frac{4x^2}{x^2+5} - 2\ln(x^2+5) \right\}$$