Section 4.8

2.) $f(x) = \sqrt{x} \xrightarrow{D} f'(x) = \frac{1}{2\sqrt{x}}$, $a = 36 \longrightarrow$
$L(x) = f(a) + f'(a)(x-a) = \sqrt{36} + \frac{1}{2\sqrt{36}}(x-36) \longrightarrow$
$L(x) = 6 + \frac{1}{12}x - 3 \longrightarrow$ $\boxed{L(x) = 3 + \frac{1}{12}x}$;
since $f(x) \approx L(x)$ then $f(35) \approx L(35) \longrightarrow$
$\sqrt{35} \approx L(35) = 3 + \frac{1}{12}(35) \approx 5.9166$;
<u>calculator</u> : $\sqrt{35} \approx 5.9161$

3.) Let $f(x) = x^{1/3} \xrightarrow{D} f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$;
let $a = 125$ (convenient and close to 124) ; then
$L(x) = f(a) + f'(a)(x-a) = 125^{1/3} + \frac{1}{3(125^{2/3})}(x-125) \longrightarrow$
$L(x) = 5 + \frac{1}{75}(x-125) = 5 - \frac{5}{3} + \frac{1}{75}x \longrightarrow$
$\boxed{L(x) = \frac{1}{75}x + \frac{10}{3}}$; since $f(x) \approx L(x)$ then
$f(124) \approx L(124) \longrightarrow$ $124^{1/3} \approx \frac{124}{75} + \frac{10}{3} \approx 4.987$;
<u>calculator</u> : $124^{1/3} \approx 4.987$

9.) Let $f(x) = \ln x \xrightarrow{D} f'(x) = \frac{1}{x}$ and let
$a = 1$ (convenient and close to 1.01) ;
then $L(x) = f(a) + f'(a)(x-a) = \cancel{\ln 1} + \frac{1}{1}(x-1) \longrightarrow$
$\boxed{L(x) = x - 1}$; since $f(x) \approx L(x)$ then
$f(1.01) \approx L(1.01) = 1.01 - 1 = 0.01$;
<u>calculator</u> : $\ln(1.01) \approx 0.01$

11.) $f(x) = \frac{1}{1+x} \xrightarrow{D} f'(x) = \frac{(1+x)(0) - 1(1)}{(1+x)^2} = \frac{-1}{(1+x)^2}$

and $a = 0$ so

$L(x) = f(a) + f'(a)(x-a) = 1 + (-1)(x-0) \rightarrow$

$\boxed{L(x) = 1 - x}$

17.) $f(x) = \ln(1+x) \xrightarrow{D} f'(x) = \frac{1}{1+x}$ and $a = 0$

$\rightarrow L(x) = f(a) + f'(a)(x-a) = \cancel{\ln}\overset{0}{1} + (1)(x-0) \rightarrow$

$\boxed{L(x) = x}$

22.) $f(x) = e^{2x} \xrightarrow{D} f'(x) = 2e^{2x}$ and $a = 0$

$\rightarrow L(x) = f(a) + f'(a)(x-a) = e^0 + 2e^0(x-0) \rightarrow$

$\boxed{L(x) = 1 + 2x}$

30.) $f(x) = \left(1 + \frac{1}{x}\right)^{1/4} \xrightarrow{D} f'(x) = \frac{1}{4}\left(1 + \frac{1}{x}\right)^{-3/4} \cdot \frac{-1}{x^2} \rightarrow a = 1 \rightarrow$

$L(x) = f(1) + f'(1)(x-1) = 2^{1/4} + \frac{-1}{4}(2)^{-3/4}(x-1) \rightarrow$

$L(x) = 2^{1/4} + \frac{-1}{2^2} \cdot \frac{1}{2^{3/4}}(x-1) = 2^{1/4} - \frac{1}{2^{11/4}}(x-1)$

32.) $\frac{1}{N} \cdot \frac{dN}{dt} = 0.02 \rightarrow \frac{dN}{dt} = 0.02N$ and

$t = 2$, $N = 50$ so let $a = 2$ so

$L(t) = N(a) + N'(a)(t-a) = N(2) + \frac{dN}{dt}(2) \cdot (t-2)$

$= 50 + 0.02(50)(t-2) = 50 + (1)(t-2) \rightarrow$

$\boxed{L(t) = 48 + t}$; since $N(t) \approx L(t)$, then

$N(2.1) \approx L(2.1) = 48 + (2.1) = 50.1$

37.) $f(x) = 3x^2 \xrightarrow{D} f'(x) = 6x$ and $x = 2 \pm 0.1$
so $a = 2$ and $\Delta x = \pm 0.1$; then
$\Delta f \approx df = f'(2)\cdot \Delta x = (12)(\pm 0.1) = \pm 1.2$,
and $f(2) = 3(2)^2 = 12$ so
$12 - 1.2 \leq f(x) \leq 12 + 1.2 \longrightarrow$
$10.8 \leq f(x) \leq 13.2$

38.) $f(x) = \sqrt{x} \xrightarrow{D} f'(x) = \frac{1}{2\sqrt{x}}$ and $x = 10 \pm 0.5$
so $a = 10$ and $\Delta x = \pm 0.5$; then
$\Delta f \approx df = f'(a)\cdot \Delta x = \frac{1}{2\sqrt{10}}\cdot(\pm 0.5) = \frac{\pm 1}{4\sqrt{10}}$,
and $f(10) = \sqrt{10}$ so
$\sqrt{10} - \frac{1}{4\sqrt{10}} \leq f(x) \leq \sqrt{10} + \frac{1}{4\sqrt{10}}$.

41.) assume $\frac{|\Delta x|}{x} \leq 2\%$; $f(x) = 4x^3 \xrightarrow{D}$
$f'(x) = 12x^2$; then max. abs. % error
in f is given by
$$\frac{|\Delta f|}{f(x)} \approx \frac{|df|}{f(x)} = \frac{|f'(x)\cdot \Delta x|}{f(x)} = \frac{|12x^2 \cdot \Delta x|}{4x^3}$$
$$= \frac{12x^2 \cdot |\Delta x|}{4x^3} = 3\cdot \frac{|\Delta x|}{x} \leq 3(2\%) = 6\%$$

43.) assume $\frac{|\Delta x|}{x} \leq 2\%$; $f(x) = \ln x \xrightarrow{D}$
$f'(x) = \frac{1}{x}$; then max. abs. % error
in f is given by

$$\frac{|\Delta f|}{f(x)} \approx \frac{|df|}{f(x)} = \frac{|f'(x)\cdot \Delta x|}{f(x)} = \frac{|\frac{1}{x}\cdot \Delta x|}{\ln x}$$

$$= \frac{1}{\ln x}\cdot \frac{|\Delta x|}{x} \underset{x=20}{\leq} \frac{1}{\ln 20}\cdot (2\%) \approx 0.668\ \%$$

44.) assume $\frac{|\Delta x|}{x} \leq 2\%$; $f(x) = (1+x)^{-1} \xrightarrow{D}$ $f'(x) = -(1+x)^{-2}(1)$; then max. abs. % error in f is given by

$$\frac{|\Delta f|}{f(x)} \approx \frac{|df|}{f(x)} = \frac{|f'(x)\cdot \Delta x|}{f(x)} = \frac{|-(1+x)^{-2}\cdot \Delta x|}{1/(1+x)}$$

$$= \frac{\frac{1}{(1+x)^2}\cdot |\Delta x|}{\frac{1}{1+x}} = \frac{1}{(1+x)^2}\cdot |\Delta x| \cdot (1+x)$$

$$= \frac{1}{1+x}\cdot |\Delta x| = \frac{x}{1+x}\cdot \frac{|\Delta x|}{x} \underset{x=4}{\leq} \frac{4}{5}(2\%)$$

$$= 1.6\ \%$$

45.) $V = \frac{4}{3}\pi r^3$ and assume $\frac{|\Delta r|}{r} \leq 3\%$; then max. abs. % error in volume is

$$\frac{|\Delta V|}{V} \approx \frac{|dV|}{V} = \frac{|V'\cdot \Delta r|}{V} = \frac{|\cancel{\frac{4}{3}\pi}\cdot 3r^2\cdot \Delta r|}{\cancel{\frac{4}{3}\pi} r^3}$$

$$= 3\cdot \frac{|\Delta r|}{r} \leq 3\cdot (3\%) = 9\%$$

46.) $V = cR^2$ and $\frac{|\Delta R|}{R} \le 5\%$; then max. abs. % error in speed is

$$\frac{|\Delta V|}{V} = \frac{|V' \cdot \Delta R|}{V} = \frac{|2\cancel{c}R \cdot \Delta R|}{\cancel{c}R^2}$$

$$= 2 \cdot \frac{|\Delta R|}{R} \le 2(5\%) = 10\%$$