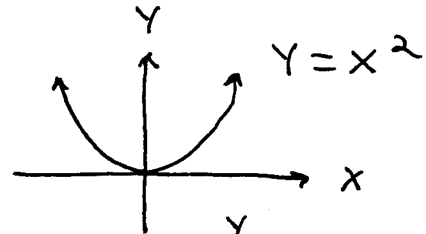
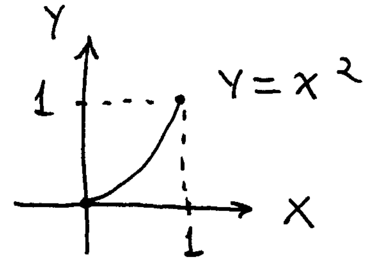
Section 1.2

1.) $f(x)=x^2$;
Range: $Y \geq 0$

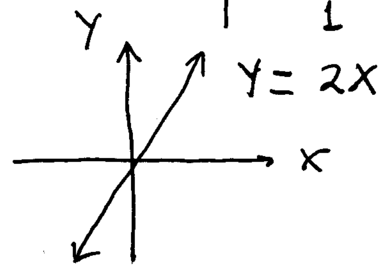


2.) $f(x)=x^2$ for $0 \leq x \leq 1$;
Range: $0 \leq Y \leq 1$



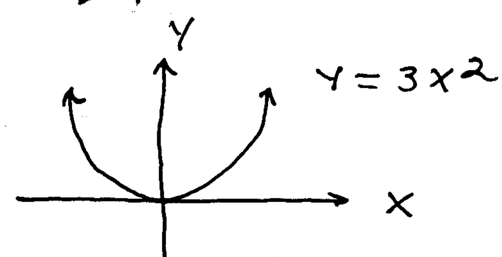
7.) $f(x)=2x$ is odd ;
Show $f(-x)=-f(x)$:

$$f(-x)=2(-x) = -2x = -f(x)$$



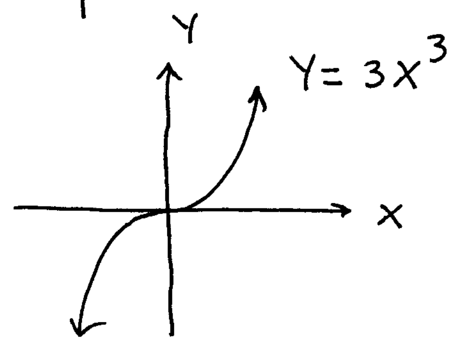
8.) $f(x)=3x^2$ is even ;
Show $f(-x)=f(x)$:

$$f(-x)=3(-x)^2 = 3x^2 = f(x)$$



12.) $f(x)=3x^3$ is odd ;
Show $f(-x)=-f(x)$:

$$f(-x)=3(-x)^3 = 3\cdot(-x^3) = -3x^3 = -f(x)$$



16.) $f(x)=\dfrac{1}{x+1}$, $x \neq -1$ and $g(x)=2x^2$

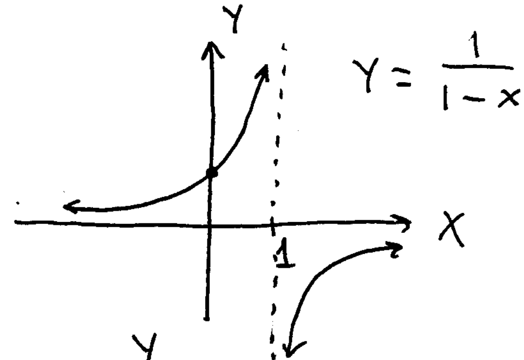
a.) $(f\circ g)(x)=f(g(x))=f(2x^2)=\dfrac{1}{2x^2+1}$;
Domain: all x-values

b.) $(g\circ f)(x)=g(f(x))=g\left(\dfrac{1}{x+1}\right)=2\cdot\left(\dfrac{1}{x+1}\right)^2$
$=\dfrac{2}{(x+1)^2}$; Domain: all $x \neq -1$

31.)

Dist	Time
1 m.	1 hr.
2 m.	2 hr.
3 m.	3 hr.

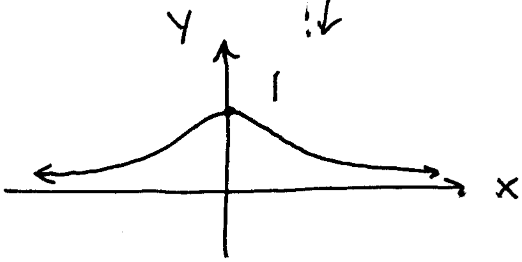
So distance $D = T$ where T is time in hours



33.) $f(x) = \dfrac{1}{1-x}$:

Domain : all $x \neq 1$

Range : all $Y \neq 0$



36.) $f(x) = \dfrac{1}{x^2+1}$:

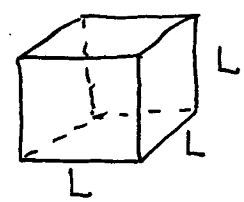
Domain : all x-values

Range : $0 < Y \leq 1$

56.) $M = 0.35V \rightarrow M = \frac{35}{100}V$

$\rightarrow V = \frac{20}{7}M$; and $V = L^3$

$\rightarrow L = V^{1/3} = \left(\frac{20}{7}M\right)^{1/3}$, i.e., $L = \left(\frac{20}{7}M\right)^{1/3}$



58.) $N = 40 \cdot 2^t$

a.) If $t = 0$, then $N = 40 \cdot 2^0 = 40 \cdot 1 = 40$

b.) (Recall : $e^{\ln z} = z$)

$N = 40 \cdot 2^t = 40 \cdot e^{\ln 2^t} = 40 \cdot e^{t \cdot \ln 2}$

c.) If $N = 1000$, then $1000 = 40 \cdot 2^t \rightarrow$

$25 = 2^t \rightarrow \ln 25 = \ln 2^t \rightarrow$

$\ln 25 = t \cdot \ln 2 \rightarrow t = \dfrac{\ln 25}{\ln 2} \approx 4.64$

59.) $N = Ce^{kt}$ and $C = 20$ mg $\rightarrow$
$N = 20e^{kt}$; $t = 5730 \rightarrow N = \frac{1}{2}C = 10$ mg
$\rightarrow 10 = 20e^{k(5730)} \rightarrow \frac{1}{2} = e^{5730k} \rightarrow$
$\ln(1/2) = \ln e^{5730k} = 5730k \rightarrow$
$k = \frac{1}{5730}\ln(1/2) \rightarrow N = 20e^{\frac{1}{5730}\ln(\frac{1}{2})\cdot t}$;
if $t = 2000 \rightarrow N = 20e^{\frac{1}{5730}\ln(\frac{1}{2})\cdot(2000)}$
≈ 15.7 mg.

60.) (SEE solution for 59.)
$N = Ce^{kt} \rightarrow N = 20e^{kt}$ and
$k = \frac{1}{5730}\ln(1/2) \rightarrow N = 20e^{\frac{1}{5730}\ln(1/2)\cdot t}$;

a.) $N = 10 \rightarrow t = 5730$ yrs. (1 $\frac{1}{2}$-life)

b.) $N = 5 \rightarrow t = 11,460$ yrs. (2 $\frac{1}{2}$-lives)

c) $N = 1 \rightarrow 1 = 20e^{\frac{1}{5730}\ln(\frac{1}{2})\cdot t} \rightarrow$
$\ln\left(\frac{1}{20}\right) = \ln e^{\frac{1}{5730}\ln(\frac{1}{2})t} \rightarrow$
$\ln\left(\frac{1}{20}\right) = \frac{1}{5730}\ln\left(\frac{1}{2}\right)\cdot t \rightarrow$

$$t = \frac{\ln(1/20) \cdot 5730}{\ln(1/2)} \approx 24{,}765 \text{ yrs.}$$

62.) $N = Ce^{kt}$ and $t = 5 \rightarrow N = 0.37C$

$\rightarrow 0.37\cancel{C} = \cancel{C}e^{5k} \rightarrow \ln 0.37 = \ln e^{5k} \rightarrow$

$\ln 0.37 = 5k \rightarrow k = \frac{1}{5}\ln 0.37 \rightarrow$

$N = Ce^{\frac{1}{5}\ln 0.37 \cdot t}$; if $N = \frac{1}{2}C$, then

$\frac{1}{2}\cancel{C} = \cancel{C}e^{\frac{1}{5}\ln 0.37 \cdot t} \rightarrow \ln\left(\frac{1}{2}\right) = \ln e^{\frac{1}{5}\ln 0.37 \cdot t}$

$\rightarrow \ln\left(\frac{1}{2}\right) = \frac{1}{5}\ln 0.37 \cdot t \rightarrow \frac{1}{2}$-life is

$$t = \frac{5\ln(1/2)}{\ln 0.37} \approx 3.486 \text{ days}$$

64.) (SEE solution for 59.) $N = Ce^{kt}$

and $k = \frac{1}{5730}\ln(1/2) \rightarrow N = Ce^{\frac{1}{5730}\ln\left(\frac{1}{2}\right) \cdot t}$;

if $N = 0.35C$, then $0.35\cancel{C} = \cancel{C}e^{\frac{1}{5730}\ln\left(\frac{1}{2}\right) \cdot t} \rightarrow$

$\ln 0.35 = \ln e^{\frac{1}{5730}\ln\left(\frac{1}{2}\right) \cdot t} = \frac{1}{5730}\ln\left(\frac{1}{2}\right)t$

$$\rightarrow t = \frac{5730\ln 0.35}{\ln(1/2)} \approx 8679 \text{ years ago}$$

65.) (SEE solution for 59.)

$N = Ce^{kt}$ and $k = \frac{1}{5730} \ln(1/2) \rightarrow$

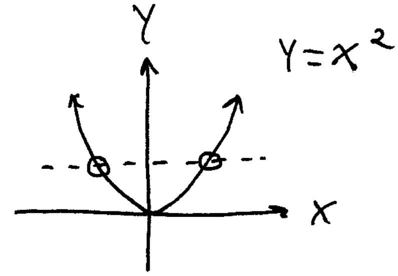
$N = Ce^{\frac{1}{5730} \ln(1/2) t}$; if $t = 15{,}000$,

then $N = Ce^{\frac{1}{5730} \ln(1/2) \cdot (15{,}000)}$

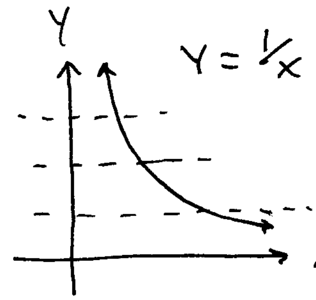
$\approx 0.163\, C$

$= 16.3\%$ of the original amount

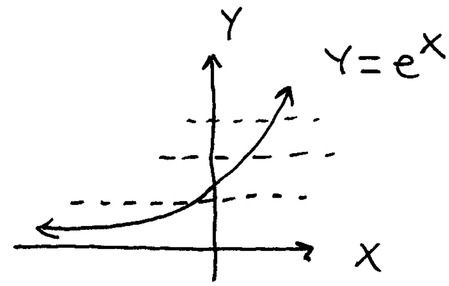
69.) b.) $f(x) = x^2$ is NOT 1-1 since it fails horizontal line test



c.) $f(x) = \frac{1}{x}$, $x > 0$ is 1-1 since it passes horizontal line test



d.) $f(x) = e^x$ is 1-1 since it passes horizontal line test

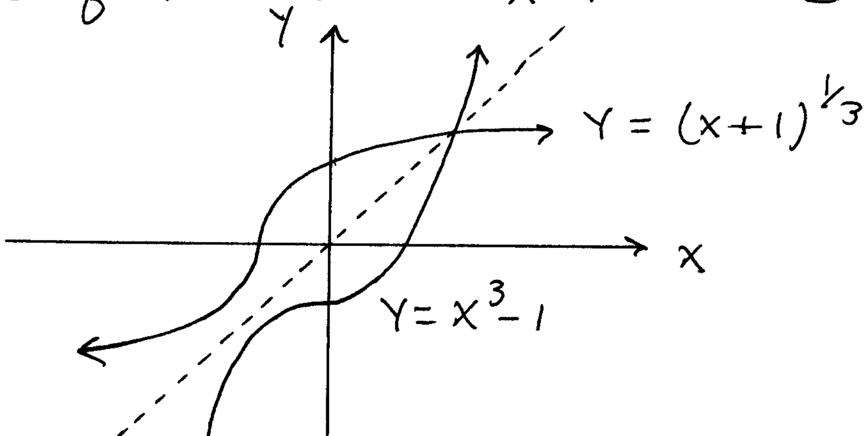


70.) a.) Let $f(x) = x^3 - 1$; show f is 1-1:

$f(x_1) = f(x_2) \rightarrow x_1^3 - 1 = x_2^3 - 1 \rightarrow x_1^3 = x_2^3 \rightarrow (x_1^3)^{1/3} = (x_2^3)^{1/3} \rightarrow x_1 = x_2$;

$Y = x^3 - 1 \rightarrow$ (switch variables) $x = Y^3 - 1 \rightarrow$ (solve for Y) $Y^3 = x + 1 \rightarrow Y = (x+1)^{1/3} \rightarrow \underline{f^{-1}(x) = (x+1)^{1/3}}$ and domain of f^{-1} is all x-values

b.)

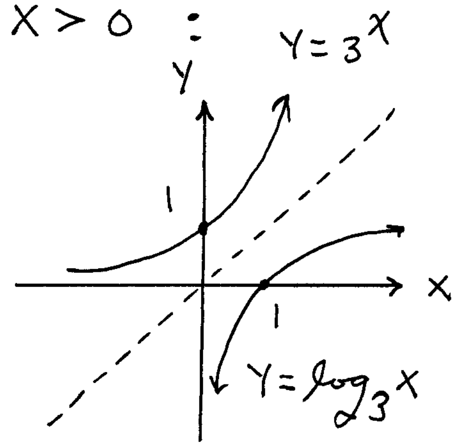


75) $Y = 3^X \rightarrow$ (switch variables)

$X = 3^Y \rightarrow$ (solve for Y)

$\log_3 X = \log_3 3^Y \rightarrow Y = \log_3 X \rightarrow f^{-1}(X) = \log_3 X$

and domain for f^{-1} is all $X > 0$:



81.) b.) $3^{4\log_3 X} = 3^{\log_3 X^4}$

$= X^4$

d.) $4^{-2\log_2 X} = 4^{\log_2 X^{-2}}$

$= (2^2)^{\log_2 X^{-2}} = 2^{2\log_2 X^{-2}}$

$= 2^{\log_2 X^{-4}} = X^{-4}$

e.) $2^{3\log_{1/2} X} = 2^{\log_{1/2} X^3} = \left(\frac{1}{\frac{1}{2}}\right)^{\log_{1/2} X^3}$

$= \dfrac{1^{\log_{1/2} X^3}}{\left(\frac{1}{2}\right)^{\log_{1/2} X^3}} = \dfrac{1}{X^3}$

82.) a.) $\log_4 16^X = \log_4 (4^2)^X = \log_4 4^{2X} = 2X$

d.) $\log_{1/2} 4^X = \log_{1/2} (2^2)^X = \log_{1/2} \left(\frac{1}{2^{-2}}\right)^X$

$= \log_{1/2} \left(\left(\frac{1}{2}\right)^{-2}\right)^X = \log_{1/2} \left(\frac{1}{2}\right)^{-2X} = -2X$

f.) $\log_3 9^{-X} = \log_3 (3^2)^{-X} = \log_3 3^{-2X} = -2X$

83.) c.) $\ln(x^2-1) - \ln(x+1) = \ln \frac{x^2-1}{x+1}$

$= \ln \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = \ln(x-1)$

84.) d.) $e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$

99.) $f(x) = 4 \sin 2\pi x$;
amplitude $= 4$;
$0 \le 2\pi x \le 2\pi \longrightarrow 0 \le x \le 1$ so
period $= 1$.