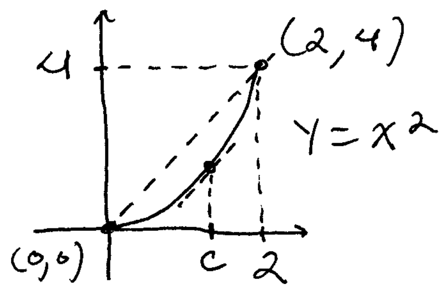
Section 5.1

35.)

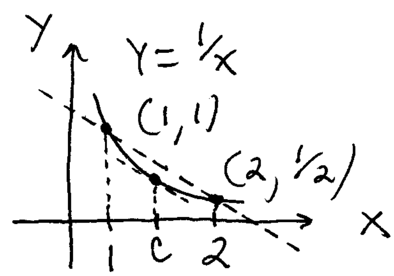


a.) Slope
$$m=\frac{f(2)-f(0)}{2-0}=\frac{4-0}{2}=2$$

b.) $f(x)=x^2$ is continuous on $[0,2]$ since it is a polynomial; $f'(x)=2x$ so f is differentiable on $(0,2)$; by MVT ~~there~~ is a # c, $0<c<2$, satisfying $\frac{f(2)-f(0)}{2-0}=f'(c) \rightarrow$

$2=2c \rightarrow c=1$

36.)



a.) Slope
$$m=\frac{f(2)-f(1)}{2-1}=\frac{\frac{1}{2}-1}{1}=\frac{-1}{2}$$

b.) $f(x)=\frac{1}{x}$ is continuous on $[1,2]$ (quotient of continuous functions and denominator $\neq 0$.); $f'(x)=\frac{-1}{x^2}$ so f is differentiable on $(1,2)$; by MVT there is a # c, $1<c<2$, satisfying

$$\frac{f(2)-f(1)}{2-1}=f'(c) \rightarrow \frac{-1}{2}=\frac{-1}{c^2} \rightarrow c^2=2$$

$$\rightarrow c=\pm\sqrt{2} \rightarrow c=+\sqrt{2}$$

38.) Let $f(x)=x^2-x-2$ on $[-1,2]$;

f is continuous on $[-1,2]$ since it is a polynomial; $f'(x) = 2x-1$ so f is differentiable on $(-1,2)$; by MVT there is a # c, $-1<c<2$, satisfying

$$\frac{f(2)-f(-1)}{2-(-1)} = f'(c) \rightarrow \frac{0-0}{3} = f'(c) \rightarrow$$

$f'(c) = 0$ (This means f has a horizontal tangent line at $x=c$.)

$\rightarrow 2c-1=0 \rightarrow 2c=1 \rightarrow c=\frac{1}{2}$

40.) Let $f(x) = \dfrac{1}{1+x^2}$ and consider interval $[-1,1]$. Then f is continuous on $[-1,1]$ (quotient of continuous functions and denominator $\neq 0$); $f(x) = (1+x^2)^{-1} \xrightarrow{D}$

$f'(x) = -(1+x^2)^{-2} \cdot 2x = \dfrac{-2x}{(1+x^2)^2}$ so

f is differentiable on $(-1,1)$; by MVT there is a # c, $-1<c<1$, satisfying $\dfrac{f(1)-f(-1)}{1-(-1)} = f'(c) \rightarrow$

$$\frac{\frac{1}{2}-\frac{1}{2}}{2} = f'(c) \rightarrow \boxed{0 = f'(c)} \rightarrow$$

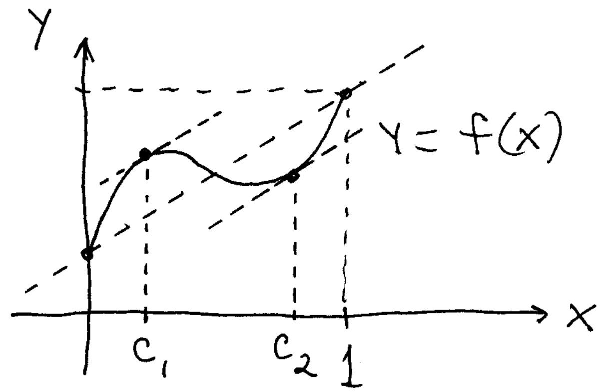
$$\frac{-2c}{(1+c^2)^2} = 0 \rightarrow -2c = 0 \rightarrow c = 0.$$

41.) Let $f(x) = -x^2 + 2$ on $[-1, 2]$; f is continuous on $[-1, 2]$ since it is a polynomial; $f'(x) = -2x$ so f is differentiable on $(-1, 2)$; by MVT ~~there is a # c, $-1<c<2$, so that~~

$$\frac{f(2) - f(-1)}{2 - (-1)} = f'(c) \rightarrow \frac{-2-1}{3} = f'(c) \rightarrow$$

$$\boxed{f'(c) = -1} \rightarrow -2c = -1 \rightarrow c = \tfrac{1}{2}$$

44.)



By MVT there is at least one # c, $0<c<1$, satisfying

$$\frac{f(1) - f(0)}{1 - 0} = f'(c) \longleftarrow \text{SLOPE of tangent line}$$

↑ SLOPE of secant line

48.) $s(t) = \frac{1}{10} t^2$ for $0 \leq t \leq 10$;

a.) ARC on $[0, 10]$ is (average velocity)

$$\frac{s(10) - s(0)}{10 - 0} = \frac{10 - 0}{10} = 1 \text{ m./sec.}$$

b.) Instantaneous velocity is $s'(t) = \frac{1}{5} t$ m./sec.

c.) If $s'(t) = 1 \to \frac{1}{5}t = 1 \to t = 5$ sec.

55.) Assume that $|f(x) - f(y)| \leq |x-y|^2$.
Show that $f(x) = C$ for some constant C : (Let x, y be real #'s.)

$$|f(x) - f(y)| \leq |x-y|^2 \longrightarrow \frac{|f(x) - f(y)|}{|x-y|} \leq |x-y|$$

$$(\text{for } x \neq y) \longrightarrow \left| \frac{f(x) - f(y)}{x - y} \right| \leq |x-y| \longrightarrow$$

$$\lim_{y \to x} \left| \frac{f(y) - f(x)}{y - x} \right| = \lim_{y \to x} |x-y| \longrightarrow$$

$|f'(x)| = 0 \to f'(x) = 0$ for all x-values

$\to f(x) = C$ for some constant C.