Section 5.2

2.) $Y = X^2 + 5X$, Domain: all x-values

$\xrightarrow{D} Y' = 2X + 5 = 0$

$\longrightarrow 2X = -5 \longrightarrow X = \frac{-5}{2}$;

$-$ 0 $+$ ——— Y'

$\left.\begin{array}{l} X = \frac{-5}{2} \\ Y = \frac{-25}{4} \end{array}\right\}$ abs. min.

$\xrightarrow{D} Y'' = 2$

$+ \quad + \quad +$ ——— Y''

Y is ↑ for $X > \frac{-5}{2}$,

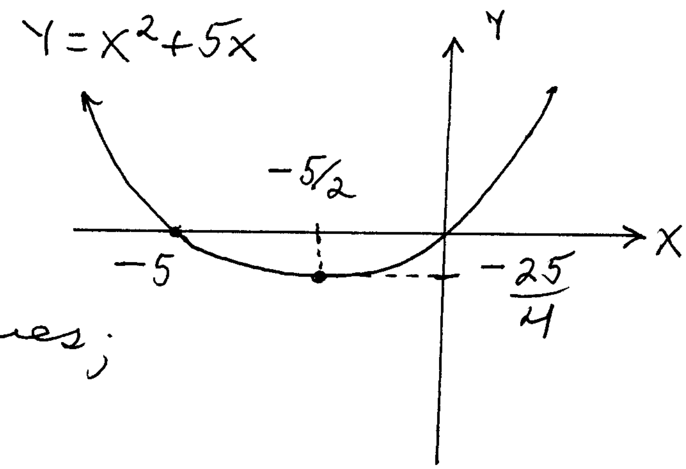
Y is ↓ for $X < -5/2$,

Y is ∪ for all x-values;

$X = 0: \ Y = 0$

$Y = 0: \ X(X+5) = 0 \longrightarrow$

$X = 0, \ X = -5$



6.) $Y = (X-2)^3 + 3$, Domain: all x-values

$\xrightarrow{D} Y' = 3(X-2)^2 \cdot (1) \stackrel{?}{=} 0$

$\longrightarrow X = 2$;

$+$ 0 $+$ ——— Y'

$X = 2$

$Y = 3$

$\xrightarrow{D} Y'' = 6(X-2)(1) = 0$

$\longrightarrow X = 2$;

$-$ 0 $+$ ——— Y''

infl. pt. $\left\{\begin{array}{l} X = 2 \\ Y = 3 \end{array}\right.$

Y is ↑ for $X < 2$, $X > 2$,

Y is ∪ for $X > 2$

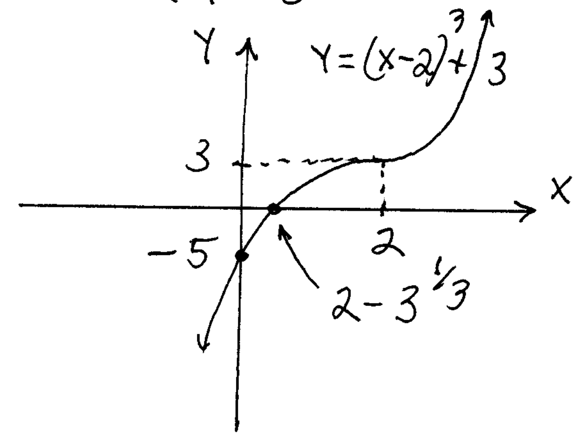
Y is ∩ for $X < 2$;

$X = 0: \ Y = -5$

$Y = 0: \ (X-2)^3 + 3 = 0$

$\rightarrow (X-2)^3 = -3 \longrightarrow X - 2 = -3^{1/3}$

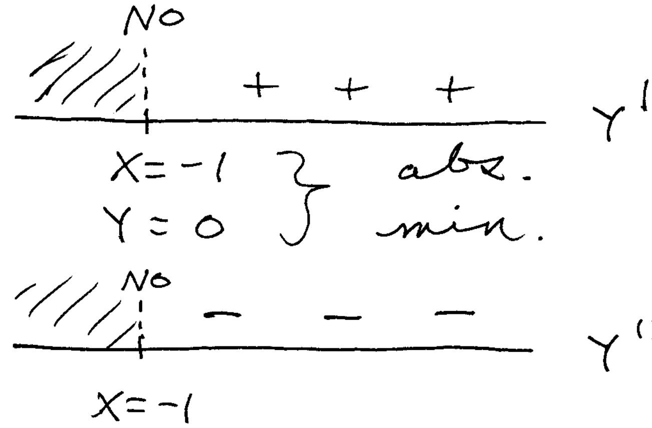
$\rightarrow X = 2 - 3^{1/3}$



7.) $Y = \sqrt{X+1}$ Domain $X \geq -1$

$\xrightarrow{D} Y' = \frac{1}{2}(X+1)^{-1/2} \cdot (1)$

$= \frac{1}{2\sqrt{X+1}}$;



$\xrightarrow{D} Y'' = \frac{-1}{4}(X+1)^{-3/2}$

$= \frac{-1}{4(X+1)^{3/2}}$;

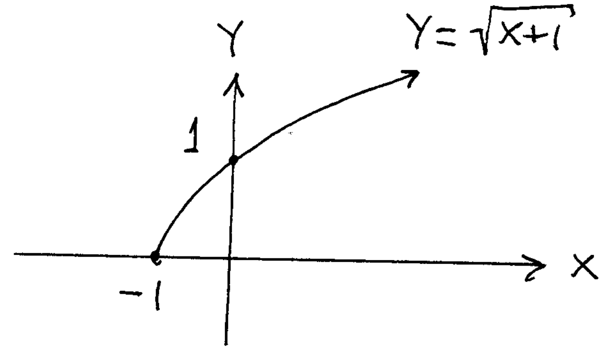
Y is ↑ for $X > -1$,

Y is ∩ for $X > -1$;

$X = 0: \ Y = 1$,

$Y = 0: \ \sqrt{X+1} = 0$

$\rightarrow X = -1$



10.) $y = \dfrac{-2}{x^2+3}$, Domain: all x-values

$\xrightarrow{D} y' = \dfrac{(x^2+3)(0)-(-2)(2x)}{(x^2+3)2} = \dfrac{4x}{(x^2+3)^2} = 0$

$\underset{}{-\ \ 0\ \ +}$ y' $\xrightarrow{D}$

abs. min. $\begin{cases} x=0 \\ y=-2/3 \end{cases}$

$y'' = \dfrac{(x^2+3)^2(4)-(4x)\cdot 2(x^2+3)\cdot 2x}{(x^2+3)^4}$

$= \dfrac{4(x^2+3)\left[(x^2+3)-4x^2\right]}{(x^2+3)^4} = \dfrac{4(3-3x^2)}{(x^2+3)^3}$

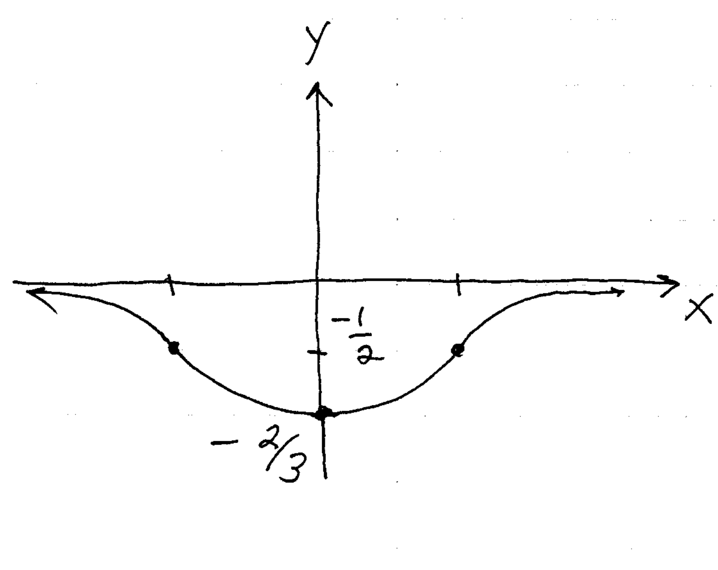
$= \dfrac{12(1-x)(1+x)}{(x^2+3)^3} = 0$ $\quad - \ 0 \ + \ 0 \ -$ y''

Infl. Pt. $\begin{cases} x=-1 \\ y=-1/2 \end{cases}$ $\quad \begin{cases} x=1 \\ y=-1/2 \end{cases}$ Infl. Pt.

y is ↑ for $x>0$,
y is ↓ for $x<0$,
y is ∪ for $-1<x<1$,
y is ∩ for $x<-1$, $x>1$;
$x=0$: $y=-2/3$
$y=0$: impossible

$\lim\limits_{x\to\pm\infty} \dfrac{-2}{x^2+3} = \dfrac{-2}{\infty} = 0$

so H.A. is $y=0$



14.) $y = \dfrac{x^2}{x^2+1}$ Domain: all x-values (restriction $x \geq 0$ given!)

$$\xrightarrow{D} y' = \frac{(x^2+1)\cdot 2x - x^2 \cdot 2x}{(x^2+1)^2} = \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2} \rightarrow$$

$$y' = \frac{2x}{(x^2+1)^2} = 0$$

0 + + + f'

$x=0 \leftarrow$ abs. min.

$$\xrightarrow{D} y'' = \frac{(x^2+1)^2(2) - 2x\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

$$= \frac{2\cancel{(x^2+1)}\cdot[(x^2+1) - 4x^2]}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{2(1-3x^2)}{(x^2+1)^3} = 0$$

+ 0 − f''

$x=0$ $\left.\begin{array}{l} x = 1/\sqrt{3} \\ y = 1/4 \end{array}\right\}$ infl. pt.

$$\rightarrow x^2 = \frac{1}{3} \rightarrow x = \frac{\pm 1}{\sqrt{3}}$$

$x = 0:\ y = 0$

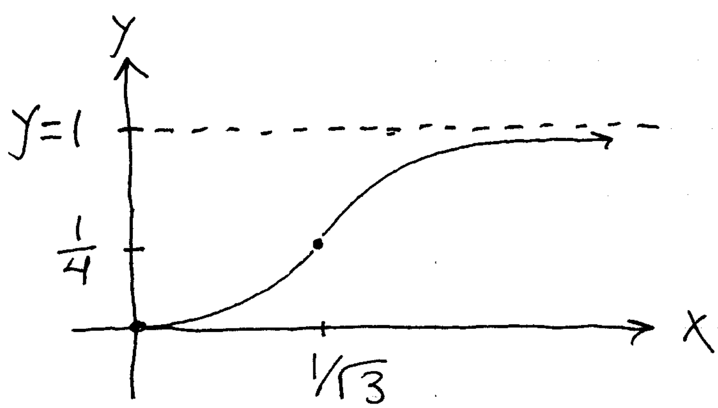
$y = 0:\ x = 0$

$$\lim_{x\to\pm\infty} \frac{x^2}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\pm\infty} \frac{1}{1+1/x^2}$$

$$= \frac{1}{1+0} = 1, \text{ so}$$

H.A. is $y=1$;

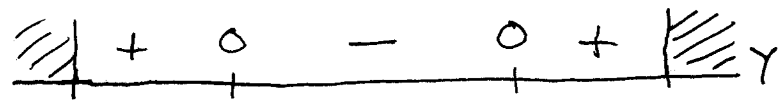


y is ↑ for $x>0$,
y is ∪ for $0<x<1/\sqrt{3}$,
y is ∩ for $x> 1/\sqrt{3}$

15.) $Y = \sin X$, Domain: $0 \le X \le 2\pi$

$\xrightarrow{D} Y' = \cos X = 0$

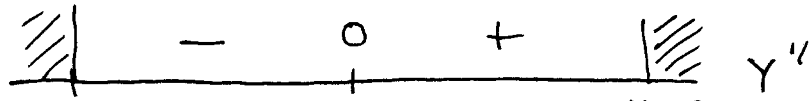
$\rightarrow X = \frac{\pi}{2}, \frac{3\pi}{2}$;



$\underbrace{X=0, Y=0}_{\text{rel. min.}}$ $\underbrace{X=\frac{\pi}{2}, Y=1}_{\text{abs. max.}}$ $\underbrace{X=\frac{3\pi}{2}, Y=-1}_{\text{abs. min.}}$ $\underbrace{X=2\pi, Y=0}_{\text{rel. max.}}$

$\xrightarrow{D} Y'' = -\sin X = 0$

$\rightarrow X = 0, \pi, 2\pi$;



$X=0$ $\quad \left.\begin{matrix} X=\pi \\ Y=0 \end{matrix}\right\}$ infl. pt. $\quad X = 2\pi$

Y is ↓ for $\frac{\pi}{2} < X < \frac{3\pi}{2}$,

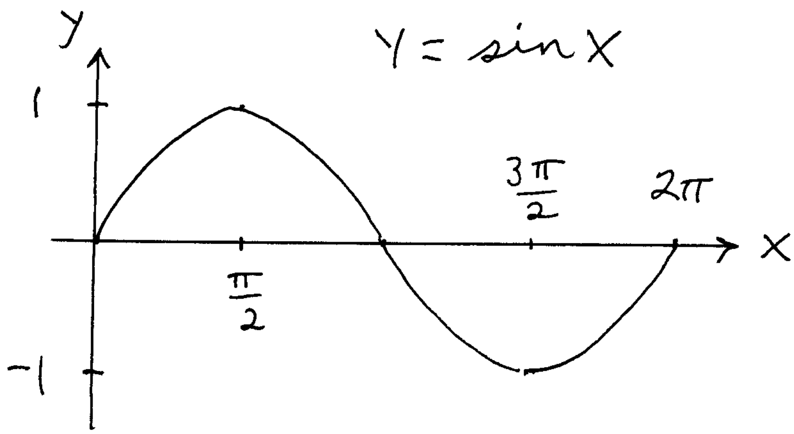
Y is ↑ for $0 < X < \frac{\pi}{2}$, $\frac{3\pi}{2} < X < 2\pi$,

Y is ∪ for $\pi < X < 2\pi$,

Y is ∩ for $0 < X < \pi$;

$X=0: \; Y=\sin 0 = 0$

$Y=0: \; \sin X = 0 \longrightarrow X = 0, \pi, 2\pi$



17.) $Y=e^X$, Domain: all x-values

$\xrightarrow{D} Y'=e^X$,

+	+	+	Y'

$\xrightarrow{D} Y''=e^X$

+	+	+	Y''

Y is $\uparrow$ for $-\infty < X < \infty$,

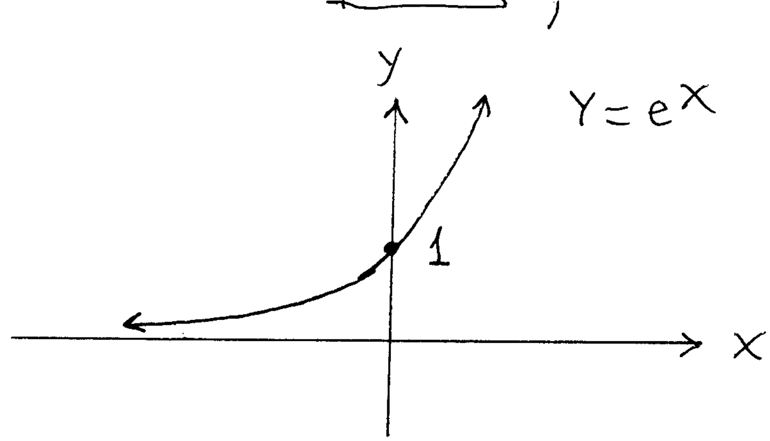
Y is $\cup$ for $-\infty < X < \infty$;

$X=0: \; Y=e^0=1$

$Y=0: \; e^X=0$ (NO);

$\lim_{X\to\infty} e^X = e^{\infty} = \infty, \; \lim_{X\to-\infty} e^X = e^{-\infty} = "\frac{1}{e^{\infty}}" = 0$

so H.A. is $\boxed{Y=0}$;



19.) $Y = e^{-\frac{x^2}{2}}$, Domain: all x-values

$\xrightarrow{D} Y' = e^{-\frac{x^2}{2}} \cdot (-x) = -x e^{-\frac{x^2}{2}} = \dfrac{-x}{e^{x^2/2}} = 0$

$\rightarrow X = 0$;

$$\begin{array}{ccc} + & 0 & - \\ \hline & X=0 & \end{array} \; Y'$$

$\left.\begin{array}{l} X = 0 \\ Y = 1 \end{array}\right\}$ abs. max.

$Y'' = D\left(\dfrac{-x}{e^{x^2/2}}\right) = \dfrac{e^{x^2/2}\cdot(-1) - (-x)\cdot e^{x^2/2}\cdot x}{(e^{x^2/2})^2}$

$= \dfrac{e^{x^2/2}\cdot(x^2-1)}{e^{x^2}} = \dfrac{(x-1)(x+1)}{e^{x^2/2}} = 0 \rightarrow$

$(x-1)(x+1) = 0 \rightarrow X = 1, X = -1$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & X=-1 & & X=1 & \end{array} \; Y''$$

$Y = e^{-1/2}$ $\quad$ $Y = e^{-1/2}$

infl. pts.

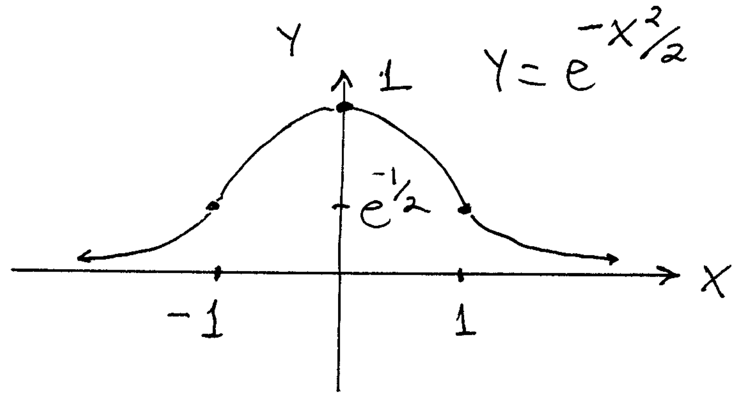
Y is ↑ for $x < 0$,
Y is ↓ for $x > 0$,
Y is ∪ for $x < -1$, $x > 1$,
Y is ∩ for $-1 < x < 1$;

$\lim_{x \to \pm\infty} e^{-\frac{x^2}{2}} = e^{-\infty} = \text{"}\dfrac{1}{e^{\infty}}\text{"} = \text{"}\dfrac{1}{\infty}\text{"} = 0$, so

H.A. is $\boxed{Y = 0}$;

$X = 0$: $Y = 1$

$Y = 0$: $e^{-\frac{x^2}{2}} = 0$ (NO)



20.) $Y = \frac{1}{1+e^{-x}} \cdot \frac{e^x}{e^x} = \frac{e^x}{e^x + e^0} = \frac{e^x}{e^x+1}$,

Domain : all x-values

$\xrightarrow{D} Y' = \frac{(e^x+1)e^x - e^x \cdot e^x}{(e^x+1)^2} = \frac{e^{2x} + e^x - e^{2x}}{(e^x+1)^2}$

$= \frac{e^x}{(e^x+1)^2} = 0 \rightarrow e^x = 0$ (No) ;

$+ \quad + \quad +$ Y'

$\xrightarrow{D} Y'' = \frac{(e^x+1)^2 \cdot e^x - e^x \cdot 2(e^x+1) \cdot e^x}{(e^x+1)^4}$

$= \frac{e^x (e^x+1) \cdot [(e^x+1) - 2e^x]}{(e^x+1)^4}$

$= \frac{e^x \cdot [1-e^x]}{(e^x+1)^3} = 0 \longrightarrow e^x [1-e^x] = 0$

$\longrightarrow e^x = 0$ (No) or

$1 - e^x = 0 \longrightarrow e^x = 1 \longrightarrow x = 0$;

Y is ↑ for $-\infty < x < \infty$,

Y is ∪ for $x < 0$,

Y is ∩ for $x > 0$;

$+ \quad 0 \quad -$ Y''

$\left.\begin{array}{l} x=0 \\ Y=\frac{1}{2} \end{array}\right\}$ infl. pt.

$x = 0 : Y = 1/2$

$Y = 0 : e^x = 0$ (No) ;

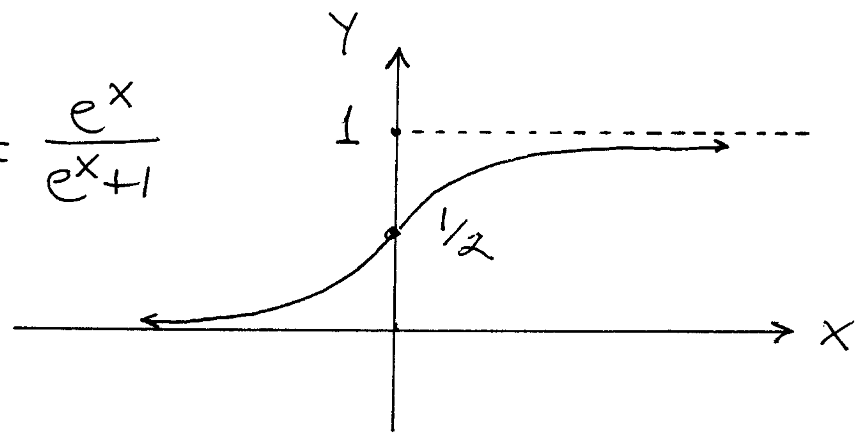
$\lim_{x\to\infty} \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e^{\infty}}} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$

so H.A. is $\boxed{Y = 1}$;

$$\lim_{x \to -\infty} \frac{1}{1+e^{-x}} = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

so H.A. is $\boxed{Y=0}$;

$$Y = \frac{e^x}{e^x+1}$$



29.) $f(N) = N\left(1-\left(\frac{N}{K}\right)^{\theta}\right) \xrightarrow{D}$

$$f'(N) = N \cdot -\theta\left(\frac{N}{K}\right)^{\theta-1} \cdot \left(\frac{1}{K}\right) + (1)\cdot\left(1-\left(\frac{N}{K}\right)^{\theta}\right)$$

$$= -\theta \cdot \frac{N \cdot N^{\theta-1}}{K^{\theta-1}K} + 1 - \left(\frac{N}{K}\right)^{\theta}$$

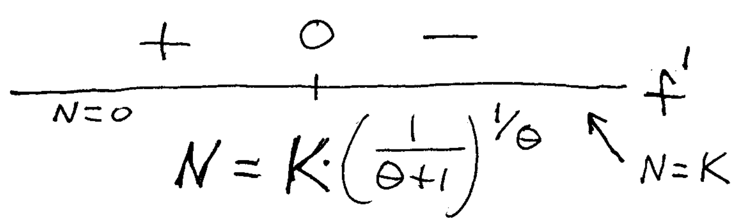
$$= -\theta \cdot \frac{N^{\theta}}{K^{\theta}} + 1 - \frac{N^{\theta}}{K^{\theta}}$$

$$= 1 - \left(\frac{N}{K}\right)^{\theta} \cdot (\theta+1) = 0 \longrightarrow$$

$$1 = \left(\frac{N}{K}\right)^{\theta}(\theta+1) \longrightarrow \left(\frac{N}{K}\right)^{\theta} = \frac{1}{\theta+1} \longrightarrow$$

$$\frac{N}{K} = \left(\frac{1}{\theta+1}\right)^{1/\theta} \longrightarrow$$

$$N = K\left(\frac{1}{\theta+1}\right)^{1/\theta}$$



f is $\uparrow$ for $0 < N < K\left(\frac{1}{\theta+1}\right)^{1/\theta}$,

f is $\downarrow$ for $N > K\left(\frac{1}{\theta+1}\right)^{1/\theta}$.

30.) $f(N) = \dfrac{aN}{k^2+N^2} \xrightarrow{D}$

$$f'(N) = \frac{(k^2+N^2)(a) - aN\cdot(2N)}{(k^2+N^2)^2}$$

$$= \frac{ak^2 + aN^2 - 2aN^2}{(k^2+N^2)^2} = \frac{ak^2 - aN^2}{(k^2+N^2)^2}$$

$$= \frac{a(k-N)(k+N)}{(k^2+N^2)^2} = 0 \longrightarrow$$

$a(k-N)(k+N) = 0 \longrightarrow N = -k$ (NO) or $N = k$

$$\begin{array}{ccccc} + & & 0 & & - \\ \hline & & N=k & & \end{array} \quad f'$$

f is $\uparrow$ for $0 < N < k$,
f is $\downarrow$ for $N > k$.

33.) $Y = 117\, e^{-10/x}$ for $x > 0$

a.) $Y' = 117 \cdot e^{-10/x} \cdot \dfrac{10}{x^2} > 0$ for $x > 0$

so Y is $\uparrow$ for all x-values;

$$\lim_{x\to\infty} 117\, e^{-10/x} = 117 \cdot e^0 = 117(1) = 117$$

so limiting value is $Y = 117$ ft.

b.) $Y' = \dfrac{1170\, e^{-10/x}}{x^2} \xrightarrow{D}$

$$Y'' = \frac{\cancel{x^2} \cdot 1170 \cdot e^{-10/x} \cdot \frac{10}{\cancel{x^2}} - 1170 \cdot e^{-10/x} \cdot 2x}{x^4}$$

$$= \frac{1170 \cdot (2) \cdot e^{-\frac{10}{x}} \cdot [5 - x]}{x^4} = 0 \rightarrow x = 5 \text{ yrs.}$$

$$\begin{array}{ccccc} + & & 0 & & - \\ \hline & & x=5 & & \end{array} \qquad Y''$$

Y is $\cup$ for $0 < x < 5$,
Y is $\cap$ for $x > 5$.

42.) $pH = -\log\,[H^+] \xrightarrow{D}$

$(pH)' = -\dfrac{1}{[H^+]} \cdot \dfrac{1}{\ln 10} < 0$ so

pH is $\downarrow$ as $[H^+]$ increases