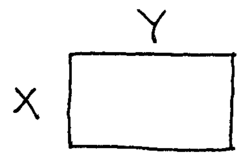
Section 5.4

1.) 

area $XY = 25 \to \boxed{Y = \frac{25}{X}}$;

minimize perimeter

$P = 2X + 2Y = 2X + 2\left(\frac{25}{X}\right) = 2X + \frac{50}{X} \to$

$\boxed{P = 2X + \frac{50}{X}} \xrightarrow{D} P' = 2 - \frac{50}{X^2} = 0 \to$

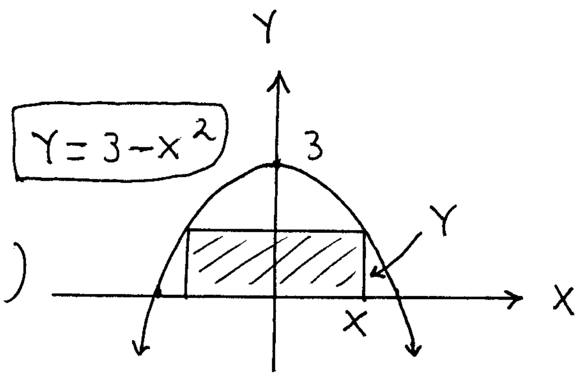
$2 = \frac{50}{X^2} \to X^2 = 25 \to X = 5$ in.

$-\ \ 0\ \ +$ P'

$X = 5$ in.

$Y = 5$ in.

min $P = 20$ in.

3.) 

maximize area

$A = (2X)Y = 2XY \to$

$A = 2X \cdot (3 - X^2) = 6X - 2X^3 \to \boxed{A = 6X - 2X^3} \xrightarrow{D}$

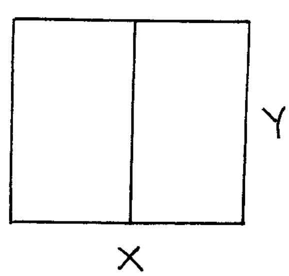
$A' = 6 - 6X^2 = 6(1 - X^2) = 0 \to X = 1$

$+\ \ 0\ \ -$ A'

$X = 1$

$Y = 2$

max. $A = 4$

4.) 

area $XY = 384 \to \boxed{Y = \frac{384}{X}}$;

minimize $L = 2X + 3Y = 2X + 3\left(\frac{384}{X}\right) \to$

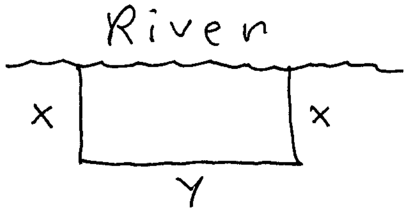
$\boxed{L = 2X + \frac{1152}{X}} \xrightarrow{D} L' = 2 - \frac{1152}{X^2} = 0 \to$

$2 = \frac{1152}{X^2} \to X^2 = 576 \to X = 24$ ft.

$-\ \ 0\ \ +$ L'

$X = 24$ ft., $Y = 16$ ft., min. $L = 96$ ft.

5.)



Length $2X+Y=320$ ft. $\rightarrow$ $\boxed{Y=320-2X}$;

maximize area $A=XY=X(320-2X) \rightarrow$

$\boxed{A=320X-2X^2} \xrightarrow{D} A'=320-4X=0 \rightarrow$

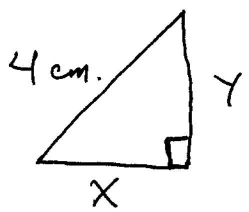
$X=80$ ft.

$$\begin{array}{ccc} + & 0 & - \\ \hline & X=80 \text{ ft.} & \end{array} \quad A'$$

$Y=160$ ft.

max. $A=12{,}800$ ft.^{2}

6.)



maximize area

$A=\frac{XY}{2}$ and $\boxed{X^2+Y^2=4^2} \rightarrow$

$Y=\sqrt{16-X^2} \longrightarrow \boxed{A=\frac{X}{2}\sqrt{16-X^2}} \xrightarrow{D}$

$$A'=\frac{X}{2}\cdot\frac{1}{\cancel{2}}(16-X^2)^{-1/2}\cdot(-\cancel{2}X)+\left(\frac{1}{2}\right)\sqrt{16-X^2}$$

$$=\frac{-X^2}{2\sqrt{16-X^2}}+\frac{\sqrt{16-X^2}}{2}=\frac{-X^2+(16-X^2)}{2\sqrt{16-X^2}}$$

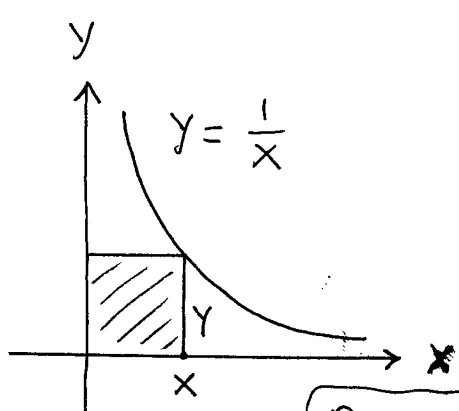
$$=\frac{16-2X^2}{2\sqrt{16-X^2}}=\frac{\cancel{2}(8-X^2)}{\cancel{2}\sqrt{16-X^2}}=0 \longrightarrow X=\sqrt{8} \text{ cm.}$$

$$\begin{array}{ccc} + & 0 & - \\ \hline & X=\sqrt{8} \text{ cm.} & \end{array} \quad A'$$

$Y=\sqrt{8}$ cm.

max. $A=4$ cm.^{2}

9.)



Minimize perimeter

$P=2X+2Y=2X+2\left(\frac{1}{X}\right) \rightarrow$

$\boxed{P=2X+\frac{2}{X}} \xrightarrow{D} P'=2-\frac{2}{X^2}=0 \rightarrow$

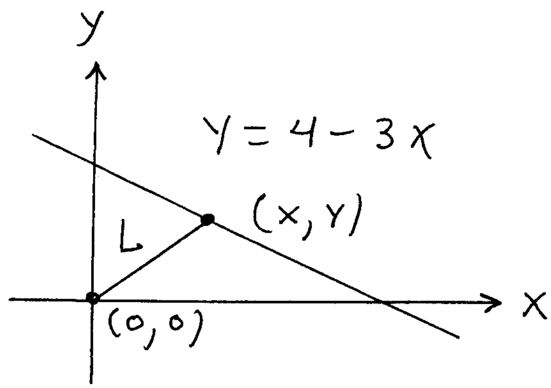
$$2 = \frac{2}{x^2} \rightarrow x^2 = 1 \rightarrow x = 1$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=1 \end{array} \quad P'$$

$x = 1$
$y = 1$

max. $P = 4$

11.)



minimize length

$$L = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\rightarrow L = \sqrt{x^2 + y^2} = \sqrt{x^2 + (4-3x)^2} \longrightarrow$$

$$\boxed{L = \sqrt{x^2 + (4-3x)^2}} \xrightarrow{D} L' = \frac{1}{2}\left(x^2 + (4-3x)^2\right)^{-1/2} \cdot \{2x + 2(4-3x) \cdot (-3)\}$$

$$= \frac{2x - 24 + 18x}{2\sqrt{x^2 + (4-3x)^2}} = \frac{20x - 24}{2\sqrt{x^2 + (4-3x)^2}} = \frac{4(5x-6)}{2\sqrt{x^2 + (4-3x)^2}} = 0$$

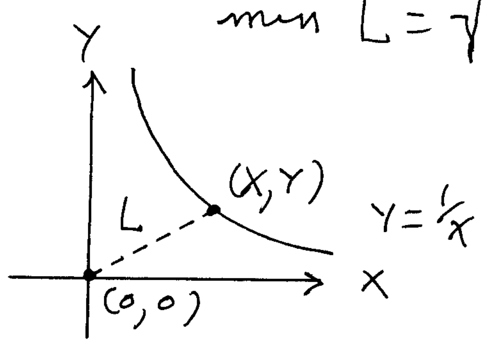
$$\rightarrow x = 6/5$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x = 6/5 \end{array} \quad L'$$

$x = 6/5$
$y = 2/5$

$$\text{min } L = \sqrt{\frac{36}{25} + \frac{4}{25}} = \sqrt{\frac{40}{25}} = \frac{2\sqrt{10}}{5}$$

13.)



minimize distance

$$L = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{1}{x}\right)^2}$$

$$\rightarrow L = \sqrt{x^2 + \frac{1}{x^2}} \xrightarrow{D}$$

$$L' = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)^{-1/2} \cdot \left\{2x - \frac{2}{x^3}\right\} = \frac{\cancel{2}\left(x - \frac{1}{x^3}\right)}{\cancel{2}\sqrt{x^2 + \frac{1}{x^2}}} = 0$$

$\rightarrow x - \frac{1}{x^3} = 0 \rightarrow x = \frac{1}{x^3} \rightarrow x^4 = 1 \rightarrow x = 1$

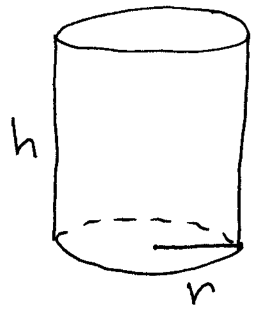
NO

$- \quad 0 \quad + \qquad L'$

$x=0 \qquad x=1$

$y = 1$

min. $L = \sqrt{2}$

17.) 

Volume $\pi r^2 h = 1000 \text{ cm}^3$

$\rightarrow \boxed{h = \frac{1000}{\pi r^2}}$;

minimize surface area

$S = S_{top} + S_{bottom} + S_{side}$

$= \pi r^2 + \pi r^2 + 2\pi r \cdot h$

$= 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right) \rightarrow$

$\boxed{S = 2\pi r^2 + \frac{2000}{r}} \xrightarrow{D} S' = 4\pi r - \frac{2000}{r^2}$

$= \frac{4\pi r^3 - 2000}{r^2} = 0 \rightarrow 4\pi r^3 - 2000 = 0 \rightarrow$

$4\pi r^3 = 2000 \rightarrow r^3 = \frac{2000}{4\pi} \rightarrow$

$r = \left(\frac{500}{\pi}\right)^{1/3} \approx 5.42$

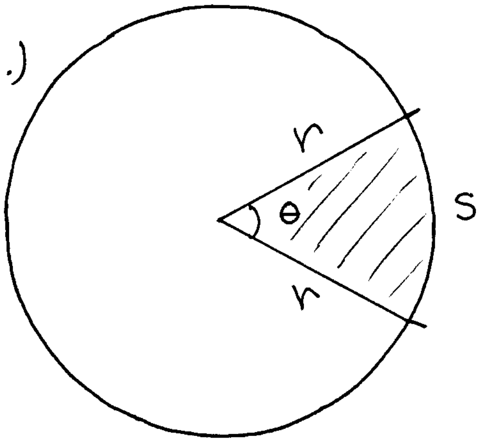
$- \quad 0 \quad + \qquad S'$

$r = \left(\frac{500}{\pi}\right)^{1/3} \approx 5.42 \text{ cm.}$

$h = \frac{1000}{\pi \left(\frac{500}{\pi}\right)^{2/3}} = \frac{500 \cdot 2}{\pi \cdot \frac{500^{2/3}}{\pi^{2/3}}} = 2\left(\frac{500}{\pi}\right)^{1/3} \approx 10.84 \text{ cm.}$

min $S = 2\pi \left(\frac{500}{\pi}\right)^{2/3} + \frac{2000}{\left(\frac{500}{\pi}\right)^{1/3}} \text{ cm.}^2$

19.) a.)



area of sector is

$$A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}\theta r^2$$

and $A = 2$ so

$$2 = \frac{1}{2}\theta r^2 \rightarrow \boxed{\theta = \frac{4}{r^2}} \;;$$

minimize perimeter

$$P = r + r + s = 2r + r\theta = 2r + r \cdot \left(\frac{4}{r^2}\right) \rightarrow$$

$$\boxed{P = 2r + \frac{4}{r}} \xrightarrow{D} P' = 2 - \frac{4}{r^2} = \frac{2r^2 - 4}{r^2} = 0$$

$$\rightarrow 2r^2 - 4 = 0 \rightarrow r^2 = 2 \rightarrow r = \sqrt{2} \;;$$

NO | $-$ | 0 | $+$ | P'

$r = 0$ | $r = \sqrt{2}$

$\theta = 2$

min. $P = 2\sqrt{2} + \frac{4}{\sqrt{2}} = 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$