Section 5.5

2.) $\lim_{x\to 2} \frac{x-2}{x^2-4} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 2} \frac{1}{2x} = \frac{1}{2(2)} = \frac{1}{4}$

3.) $\lim_{x\to -2} \frac{3x^2+5x-2}{x+2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to -2} \frac{6x+5}{1} = 6(-2)+5 = -7$

6.) $\lim_{x\to 0} \frac{3-\sqrt{2x+9}}{2x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{-\frac{1}{2}(2x+9)^{-1/2}\cdot \cancel{2}}{\cancel{2}}$

$= \frac{-1}{2}\cdot\frac{1}{\sqrt{9}} = \frac{-1}{6}$

7.) $\lim_{x\to 0} \frac{\sin x}{x\cos x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\cos x}{x\cdot -\sin x + (1)\cos x}$

$= \frac{\cos 0}{0\cdot -\sin 0 + \cos 0} = \frac{1}{1} = 1$

8.) $\lim_{x\to 0} \frac{x\sin x}{1-\cos x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{x\cos x + (1)\sin x}{-(-\sin x)}$

$= \lim_{x\to 0} \frac{x\cos x+\sin x}{\sin x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{x\cdot -\sin x + \cos x + \cos x}{\cos x}$

$= \frac{0+1+1}{1} = 2$

9.) $\lim_{x\to 0} \frac{1-\cos x}{x\tan x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{-(-\sin x)}{x\sec^2 x + \tan x}$

$= \lim_{x\to 0} \frac{\sin x}{x\sec^2 x + \tan x}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\cos x}{x\cdot 2\sec x\cdot \sec x\tan x + \sec^2 x + \sec^2 x}$

$= \frac{1}{0+(1)^2+(1)^2} = \frac{1}{2}$

11.) $\lim_{x\to 0^+} \frac{\sqrt{x}}{\ln(x+1)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x+1}} = \frac{\text{"}\frac{1}{0^+}\text{"}}{1} = \infty$

13.) $\lim_{x\to\infty} \frac{\ln(\ln x)}{x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \frac{0}{1} = 0$

14.) $\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}} = \frac{1}{\infty} = 0$

15.) $\lim_{x\to 0} \frac{2^x-1}{3^x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2^x \ln 2}{3^x \ln 3} = \frac{1\cdot\ln 2}{1\cdot \ln 3} = \frac{\ln 2}{\ln 3}$

19.) $\lim_{x\to 0} \frac{e^x-1-x}{x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x-1}{2x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x}{2}$

$= \frac{1}{2}$

20.) $\lim_{x\to 0} \frac{e^x-1-x-\frac{x^2}{2}}{x^3}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x-1-x}{3x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x-1}{6x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x}{6}$

$= \frac{1}{6}$

21.) $\lim_{x\to\infty} \frac{(\ln x)^2}{x^2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\cancel{2}(\ln x)\cdot\frac{1}{x}}{\cancel{2}x}$

$= \lim_{x\to\infty} \frac{\ln x}{x^2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$

23.) $\lim_{x\to\frac{\pi}{2}^-} \frac{\tan x}{\sec^2 x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\frac{\pi}{2}^-} \frac{\sec^2 x}{2\sec x\cdot \sec x \tan x}$

$= \lim_{x\to\frac{\pi}{2}^-} \frac{1}{2}\cot x = \lim_{x\to\frac{\pi}{2}^-} \frac{1}{2}\cdot\frac{\cos x}{\sin x}$

$= \frac{1}{2}\cdot\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{1}{2}\cdot\frac{0}{1} = 0$

24.) $\lim_{x\to 0} \frac{e^x-1}{\sin x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x}{\cos x} = \frac{e^0}{\cos 0} = \frac{1}{1} = 1$

26.) $\lim_{x\to\infty} \frac{x^2}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2}{e^x}$

$= \text{"}\frac{2}{\infty}\text{"} = 0$

30.) $\lim_{x\to 0^+} x^2 \ln x = \text{"}0\cdot\infty\text{"}$ (FLIP)

$= \lim_{x\to 0^+} \frac{\ln x}{1/x^2} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-2/x^3}$

$= \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^3}{-2} = \lim_{x\to 0^+} \frac{-1}{2}x^2 = \frac{-1}{2}(0)^2 = 0$

33.) $\lim_{x\to \frac{\pi}{2}^-} \left(\frac{\pi}{2}-x\right)\cdot\sec x = \text{"}0\cdot\infty\text{"}$ (FLIP)

$= \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\cos x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to\frac{\pi}{2}} \frac{-1}{-\sin x} = \frac{-1}{-\sin\frac{\pi}{2}} = \frac{-1}{-1} = 1$

35.) $\lim_{x\to\infty} \sqrt{x}\cdot\sin\left(\frac{1}{x}\right) = \text{"}\infty\cdot 0\text{"}$ (FLIP)

$= \lim_{x\to\infty} \frac{\sin(1/x)}{1/x^{1/2}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to\infty} \frac{\cos(1/x)\cdot -1/x^2}{-\frac{1}{2}x^{-3/2}}$

$= \lim_{x\to\infty} +2\cos\left(\frac{1}{x}\right)\cdot\frac{1}{\sqrt{x}} = 2\cos 0\cdot 0 = 2(1)(0) = 0$

37.) $\lim_{x\to 0^+} (\cot x - \csc x) = \text{"}\infty-\infty\text{"}$

$= \lim_{x\to 0^+} \left(\frac{\cos x}{\sin x} - \frac{1}{\sin x}\right) = \lim_{x\to 0^+} \frac{\cos x - 1}{\sin x}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \frac{-\sin x}{\cos x} = \frac{-\sin 0}{\cos 0} = \frac{-0}{1} = 0$

38.) $\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right) = \lim_{x\to\infty}\left(x-\sqrt{x^2\left(1-\frac{1}{x^2}\right)}\right)$

$= \lim_{x\to\infty}\left(x-\sqrt{x^2}\cdot\sqrt{1-\frac{1}{x^2}}\right) = \lim_{x\to\infty}\left(x-x\sqrt{1-\frac{1}{x^2}}\right)$

$= \lim_{x\to\infty} x\left\{1-\sqrt{1-\frac{1}{x^2}}\right\} = \text{"}\infty\cdot 0\text{"}$ (FLIP)

$= \lim_{x\to\infty}\dfrac{1-\sqrt{1-\frac{1}{x^2}}}{\frac{1}{x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to\infty}\dfrac{\cancel{+}\frac{1}{\cancel{2}}\left(1-\frac{1}{x^2}\right)^{-1/2}\cdot\frac{\cancel{2}}{x^3}}{\cancel{+}1/x^2}$

$= \lim_{x\to\infty}\dfrac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\dfrac{1}{x} = \dfrac{1}{1}\cdot(0) = 0$

40.) $\lim_{x\to 0^+}\left(\dfrac{1}{\sin^2 x}-\dfrac{1}{x}\right) = \text{"}\infty-\infty\text{"}$

$= \lim_{x\to 0^+}\dfrac{x-\sin^2 x}{x\sin^2 x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+}\dfrac{1-2\sin x\cos x}{x\cdot 2\sin x\cos x+(1)\sin^2 x}$

$= \dfrac{\text{"}1\text{"}}{0^+ + 0^+} = \dfrac{\text{"}1\text{"}}{0^+} = \infty$

42.) $\lim_{x\to 0^+} x^{\sin x} = \text{"}0^0\text{"} = \lim_{x\to 0^+} e^{\ln x^{\sin x}}$

$= \lim_{x\to 0^+} e^{\sin x\cdot\ln x} = e^{\lim_{x\to 0^+}\sin x\cdot\ln x}$

$\overset{\text{"}0\cdot\infty\text{"}}{=} e^{\lim_{x\to 0^+}\frac{\ln x}{1/\sin x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to 0^+}\frac{1/x}{\frac{-\cos x}{\sin^2 x}}}$

$= e^{\text{"}\frac{0}{-1/0^+}\text{"}} = e^{\text{"}\frac{0}{-\infty}\text{"}} = e^0 = 1$

43.) $\lim_{x\to\infty} x^{1/x} = \text{"}\infty^0\text{"} = \lim_{x\to\infty} e^{\ln x^{1/x}}$

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = e^{\lim_{x\to\infty} \frac{\ln x}{x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{1}}$$

$$= e^{0} = 1$$

44.) $$\lim_{x\to\infty} (1+e^{x})^{1/x} = \text{"}\infty^{0}\text{"} = \lim_{x\to\infty} e^{\ln(1+e^{x})^{1/x}}$$

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln(1+e^{x})} = e^{\lim_{x\to\infty} \frac{\ln(1+e^{x})}{x}}$$

$$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{e^{x}}{1+e^{x}}}{1}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{e^{x}}{e^{x}}} = e^{\lim_{x\to\infty} 1}$$

$$= e^{1} = e$$

49.) $$\lim_{x\to\infty} \left(\frac{x}{1+x}\right)^{x} = \text{"}1^{\infty}\text{"} = \lim_{x\to\infty} e^{\ln\left(\frac{x}{1+x}\right)^{x}}$$

$$= \lim_{x\to\infty} e^{x\ln\left(\frac{x}{1+x}\right)} = \text{"}\infty\cdot 0\text{"} \quad (\text{FLIP})$$

$$= \lim_{x\to\infty} e^{\frac{\ln\left(\frac{x}{1+x}\right)}{1/x}} = e^{\lim_{x\to\infty} \ln\left(\frac{x}{1+x}\right)/\frac{1}{x}}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to\infty} \frac{1}{\frac{\cancel{x}}{1+x}} \cdot \frac{(1+x)(1)-x(1)}{(1+x)^2} \Big/ \frac{-1}{\cancel{x^2}}}$$

$$= e^{\lim_{x\to\infty} \frac{-x}{x+1}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{-1}{1}} = e^{-1}$$

52.) $\displaystyle \lim_{x\to 0^+} \frac{e^x}{x} = \frac{e^0}{0^+} = \frac{1}{0^+} = +\infty$

58.) $\displaystyle \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) = \frac{1}{0^+} - \frac{1}{0^+} = \infty - \infty$

$\displaystyle = \lim_{x\to 0^+} \frac{1-\sqrt{x}}{x} = \frac{1-0}{0^+} = \frac{1}{0^+} = \infty$

60.) $\displaystyle \lim_{x\to\infty} \left(\frac{x+1}{x+2}\right)^x = 1^\infty$

$\displaystyle = \lim_{x\to\infty} e^{\ln\left(\frac{x+1}{x+2}\right)^x} = e^{\lim_{x\to\infty} x\cdot \ln\left(\frac{x+1}{x+2}\right)}$

$\displaystyle = e^{\lim_{x\to\infty} \frac{\ln\left(\frac{x+1}{x+2}\right)}{1/x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{\frac{x+1}{x+2}}\cdot\frac{(x+2)(1)-(x+1)(1)}{(x+2)^2}}{-1/x^2}}$

$\displaystyle = e^{\lim_{x\to\infty} \frac{\cancel{x+2}}{x+1}\cdot\frac{-x^2}{(x+2)^{\cancel{2}}}} = e^{\lim_{x\to\infty} \frac{-x^2}{x^2+3x+2}}$

$\displaystyle \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{-2x}{2x+3}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{-2}{2}} = e^{-1}$

61.) $\displaystyle \lim_{x\to 0} \frac{a^x-1}{b^x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{a^x \ln a}{b^x \ln b}$

$\displaystyle = \frac{a^0 \ln a}{b^0 \ln b} = \frac{\ln a}{\ln b}.$

62.) $\lim_{x\to\infty} \left(1+\frac{c}{x}\right)^x = \text{"}1^\infty\text{"}$

$= \lim_{x\to\infty} e^{\ln\left(1+\frac{c}{x}\right)^x} = e^{\lim_{x\to\infty} x\cdot\ln\left(1+\frac{c}{x}\right)}$

$\text{"}\infty\cdot 0\text{"}$

$= e^{\lim_{x\to\infty} \frac{\ln\left(1+\frac{c}{x}\right)}{1/x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1+\frac{c}{x}}\cdot\frac{\cancel{-}c}{\cancel{x^2}}}{\cancel{-}1/\cancel{x^2}}}$

$= e^{\frac{1}{1+0}\cdot c} = e^c$

66.) $\lim_{x\to 1} \frac{\sqrt{2x-x^4}-\sqrt[3]{x}}{1-\sqrt[4]{x^3}}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{\frac{1}{2}(2x-x^4)^{-1/2}\cdot(2-4x^3)-\frac{1}{3}x^{-2/3}}{-\frac{3}{4}x^{-1/4}}$

$= \frac{\frac{1}{2}(1)^{-1/2}(-2)-\frac{1}{3}(1)^{-2/3}}{-\frac{3}{4}(1)^{-1/4}} = \frac{-1-\frac{1}{3}}{-\frac{3}{4}}$

$= \frac{-4}{3}\cdot\frac{-4}{3} = \frac{16}{9}$