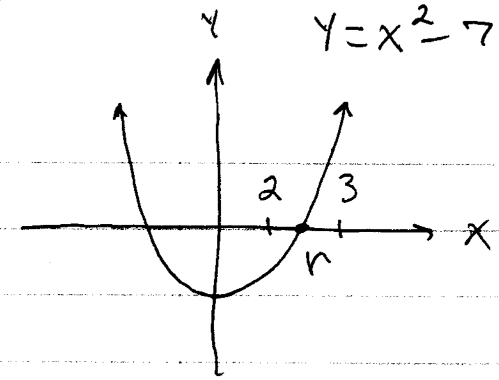
Section 5.7



1.) $x^2 - 7 = 0 \longrightarrow$

$f(x) = x^2 - 7 \xrightarrow{D}$

$f'(x) = 2x$ so

Newton: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

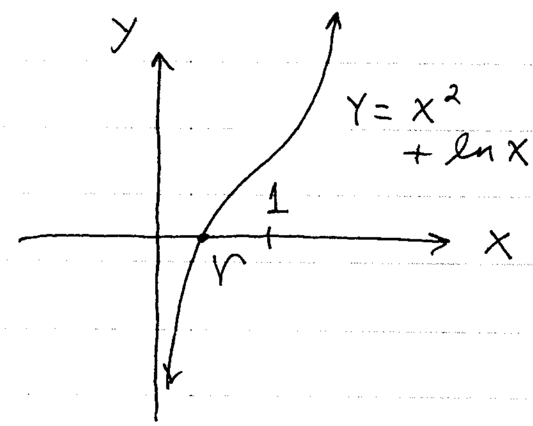
$= x_n - \frac{x_n^2 - 7}{2x_n} = \frac{2x_n^2 - x_n^2 + 7}{2x_n} \longrightarrow$

$\boxed{x_{n+1} = \frac{x_n^2 + 7}{2x_n}}$; let $x_0 = 3$ then

$x_1 = \frac{x_0^2 + 7}{2x_0} = \frac{3^2 + 7}{2(3)} \approx 2.66667$,

$x_2 = \frac{x_1^2 + 7}{2x_1} \approx \frac{(2.66667)^2 + 7}{2(2.66667)} \approx 2.64583$,

$x_3 = \frac{x_2^2 + 7}{2x_2} \approx 2.64575$, so $\boxed{r \approx 2.645}$



3.) $x^2 + \ln x = 0 \longrightarrow$

$f(x) = x^2 + \ln x \xrightarrow{D}$

$f'(x) = 2x + \frac{1}{x}$ so

Newton: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{x_n^2 + \ln x_n}{2x_n + \frac{1}{x_n}} = x_n - \frac{x_n^2 + \ln x_n}{\frac{2x_n^2 + 1}{x_n}}$

$$= \frac{x_n}{1} - \frac{x_n^3 + x_n \ln x_n}{2x_n^2 + 1}$$

$$= \frac{2x_n^3 + x_n - x_n^3 - x_n \ln x_n}{2x_n^2+1} \longrightarrow$$

$$\boxed{x_{n+1} = \frac{x_n^3 + x_n - x_n \ln x_n}{2x_n^2+1}} \; ;$$

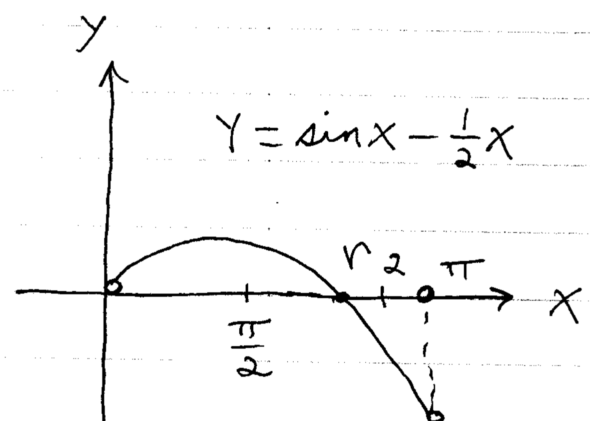
let $x_0 = 1$ then

$$x_1 = \frac{x_0^3 + x_0 - x_0 \ln x_0}{2x_0^2+1} \approx 0.66667,$$

$$x_2 = \frac{x_1^3 + x_1 - x_1 \ln x_1}{2x_1^2+1} \approx 0.65291,$$

$$x_3 = \frac{x_2^3 + x_2 - x_2 \ln x_2}{2x_2^2+1} \approx 0.65292, \text{ so}$$

root $\boxed{r \approx 0.652}$



5.) $\sin X = \frac{1}{2}X \longrightarrow$

$\sin X - \frac{1}{2}X = 0 \longrightarrow$

$f(X) = \sin X - \frac{1}{2}X \xrightarrow{D}$

$f'(X) = \cos X - \frac{1}{2}$ so

Newton: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$= \frac{x_n}{1} - \frac{\sin x_n - \frac{1}{2}x_n}{\cos x_n - \frac{1}{2}}$$

$$= \frac{x_n \cos x_n - \cancel{\tfrac{1}{2} x_n} - \sin x_n + \cancel{\tfrac{1}{2} x_n}}{\cos x_n - \tfrac{1}{2}} \longrightarrow$$

$$\boxed{x_{n+1} = \frac{x_n \cos x_n - \sin x_n}{\cos x_n - \tfrac{1}{2}}} \; ;$$

let $x_0 = 2$ then

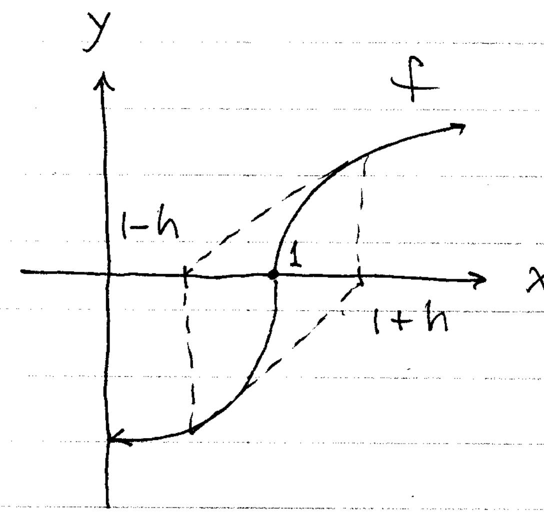
$$x_1 = \frac{x_0 \cos x_0 - \sin x_0}{\cos x_0 - \tfrac{1}{2}} \approx 1.90100 \, ,$$

$$x_2 = \frac{x_1 \cos x_1 - \sin x_1}{\cos x_1 - \tfrac{1}{2}} \approx 1.89551 \, ,$$

$$x_3 = \frac{x_2 \cos x_2 - \sin x_2}{\cos x_2 - \tfrac{1}{2}} \approx 1.89549 \, ,$$

so root $\boxed{r \approx 1.895}$

6.) $f(x) = \begin{cases} \sqrt{x-1} & \text{if } x \geq 1 \\ -\sqrt{1-x} & \text{if } x \leq 1 \end{cases}$



so $f'(x) = \begin{cases} \dfrac{1}{2\sqrt{x-1}} & \text{if } x > 1 \\ \dfrac{1}{2\sqrt{1-x}} & \text{if } x < 1 \end{cases}$;

If $\boxed{x_0 = 1 + h} > 1$ then (Newton)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{\sqrt{x_0 - 1}}{\frac{1}{2\sqrt{x_0-1}}}$$

$$= x_0 - 2(x_0 - 1) = x_0 - 2x_0 + 2$$

$$= -x_0 + 2 = -(1+h) + 2 = -1 - h + 2 = 1 - h$$

$\rightarrow \boxed{x_1 = 1 - h} < 1$; then

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{-\sqrt{1-x_1}}{\frac{1}{2\sqrt{1-x_1}}}$$

$$= x_1 + 2(1 - x_1) = x_1 - 2x_1 + 2 =$$

$$= -x_1 + 2 = -(1-h) + 2 = -1 + h + 2 = 1 + h$$

$\rightarrow \boxed{x_2 = 1 + h}$; similarly, if

$x_0 = 1 - h \rightarrow x_1 = 1 + h \rightarrow x_2 = 1 - h$;

the graph shows that we are in an infinite "loop" and the sequence of numbers determined by Newton's Method does NOT converge to r.

8.) $x^4 - x^2 = 0 \rightarrow f(x) = x^4 - x^2 \xrightarrow{D}$
$f'(x) = 4x^3 - 2x$, then

Newton : $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{x_n^4 - x_n^2}{4x_n^3 - 2x_n} = \frac{4x_n^4 - 2x_n^2 - x_n^4 + x_n^2}{4x_n^3 - 2x_n}$$

$$\rightarrow \boxed{x_{n+1} = \frac{3x_n^4 - x_n^2}{4x_n^3 - 2x_n}} \quad ; \text{ if } x_0 = \frac{-1}{2}\sqrt{2} = \frac{-1}{\sqrt{2}}$$

then

$$x_1 = \frac{3x_0^4 - x_0^2}{4x_0^3 - 2x_0} = \frac{3\left(\frac{-1}{\sqrt{2}}\right)^4 - \left(\frac{-1}{\sqrt{2}}\right)^2}{4\left(\frac{-1}{\sqrt{2}}\right)^3 - 2\left(\frac{-1}{\sqrt{2}}\right)}$$

$$= \frac{3\left(\frac{1}{4}\right) - \frac{1}{2}}{4\left(\frac{-1}{2\sqrt{2}}\right) + \frac{2}{\sqrt{2}}} = \frac{\frac{1}{4}}{0} \longrightarrow$$

$4x_0^3 - 2x_0 = f'(x_0) = 0$ (horizontal tangent line at $x = x_0 = {}^{-1}/\sqrt{2}$