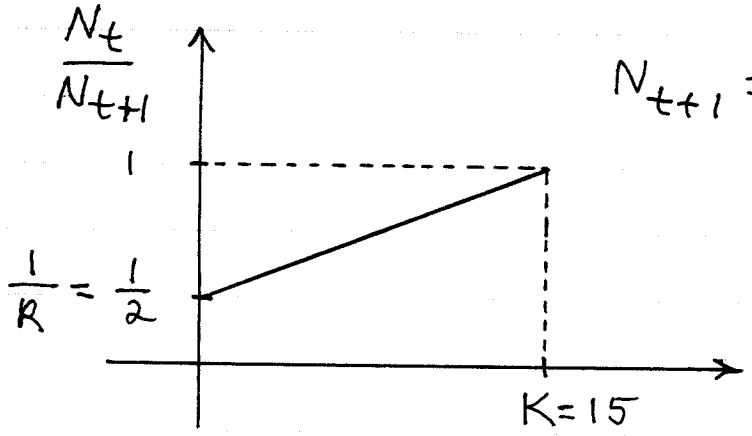
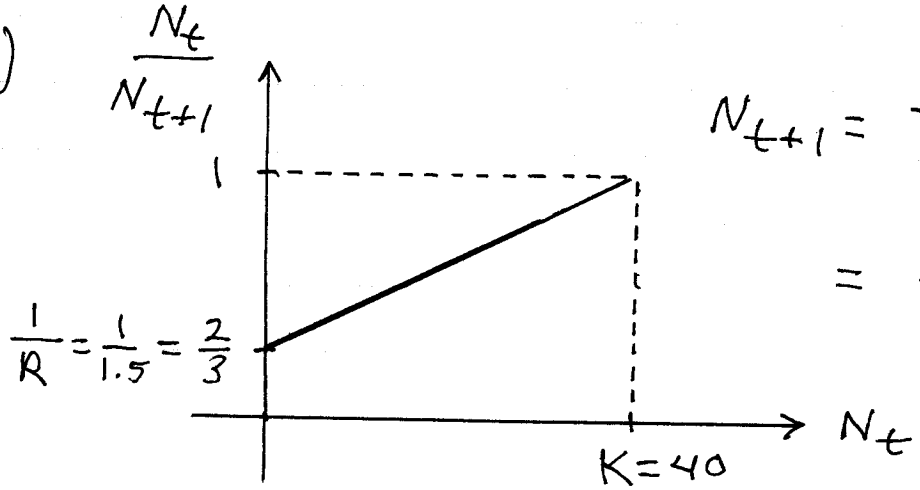
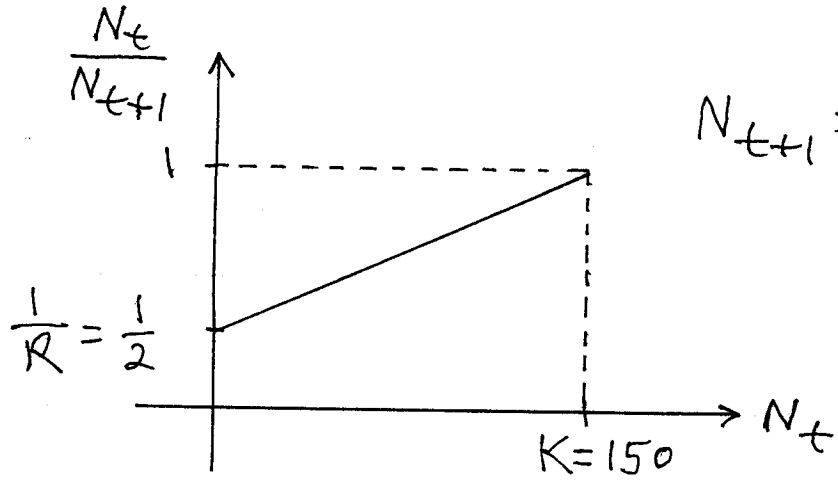
Section 2.3

1.) 

$$N_{t+1} = \frac{2N_t}{1 + \frac{1}{15}N_t}$$

3.) 

$$N_{t+1} = \frac{1.5N_t}{1 + \frac{0.5}{40}N_t} = \frac{1.5N_t}{1 + \frac{1}{80}N_t}$$

6.) 

$$N_{t+1} = \frac{2N_t}{1 + \frac{1}{150}N_t}$$

8.) $$N_{t+1} = \frac{3N_t}{1 + \frac{3-1}{40}N_t}$$ so $R=3$, $K=40$

9.) $N_{t+1} = \dfrac{1.5\,N_t}{1+\frac{1.5-1}{30}N_t}$ so $R=1.5$, $K=30$

12.) $N_{t+1} = \dfrac{5N_t}{1+\frac{1}{20}N_t} = \dfrac{5N_t}{1+\frac{5-1}{80}N_t}$

so $R=5$, $K=80$

14.) $N = \dfrac{3N}{1+\frac{1}{60}N} \rightarrow N\left(1+\frac{1}{60}N\right) = 3N \rightarrow$

$N\left(1+\frac{1}{60}N\right) - 3N = 0 \rightarrow N\left(\left(1+\frac{1}{60}N\right)-3\right)=0 \rightarrow$

$N\left(\frac{1}{60}N-2\right)=0 \rightarrow N=0$ or $N=120$

15.) $N = \dfrac{2N}{1+\frac{1}{30}N} \rightarrow N\left(1+\frac{1}{30}N\right) = 2N \rightarrow$

$N\left(1+\frac{1}{30}N\right) - 2N = 0 \rightarrow N\left[\left(1+\frac{1}{30}N\right)-2\right]=0 \rightarrow$

$N\left(\frac{1}{30}N-1\right)=0 \rightarrow N=0$ or $N=30$

20.) $N_{t+1} = \dfrac{2N_t}{1+\frac{2-1}{20}N_t} = \dfrac{2N_t}{1+\frac{1}{20}N_t}\cdot\dfrac{20}{20} = \dfrac{40N_t}{20+N_t}$;

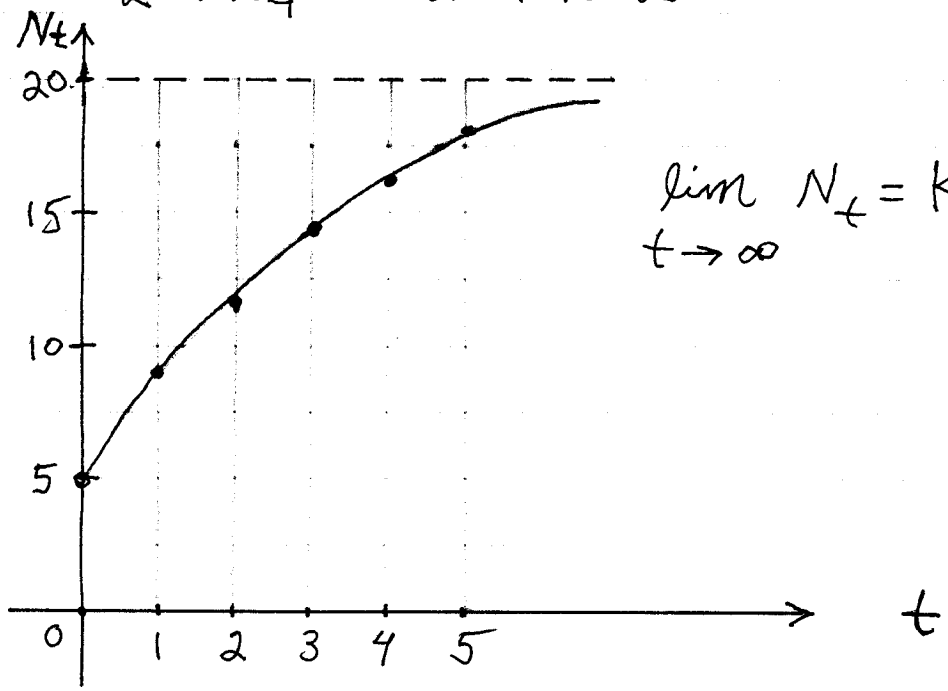
t:	0	1	2	3	4	5
N_t:	5	8	11.43	14.55	16.85	18.29

$N_1 = \dfrac{40N_0}{20+N_0} = \dfrac{40(5)}{20+5} = 8$;

$N_2 = \dfrac{40N_1}{20+N_1} = \dfrac{40(8)}{20+8} \approx 11.43$; $N_3 = \dfrac{40N_2}{20+N_2} \approx \dfrac{40(11.43)}{20+11.43} \approx 14.55$;

$$N_4 = \frac{40(N_3)}{20+N_3} \approx \frac{40(14.55)}{20+14.55} \approx 16.85\ ;$$

$$N_5 = \frac{40(N_4)}{20+N_4} \approx \frac{40(16.85)}{20+16.85} \approx 18.29$$



$$\lim_{t\to\infty} N_t = K = 20$$

21.) $$N_{t+1} = \frac{3N_t}{1+\frac{3-1}{15}N_t} = \frac{3N_t}{1+\frac{2}{15}N_t} \cdot \frac{15}{15} = \frac{45N_t}{15+2N_t}\ ;$$

t:	0	1	2	3	4	5
N_t:	1	2.65	5.87	9.88	12.79	14.18

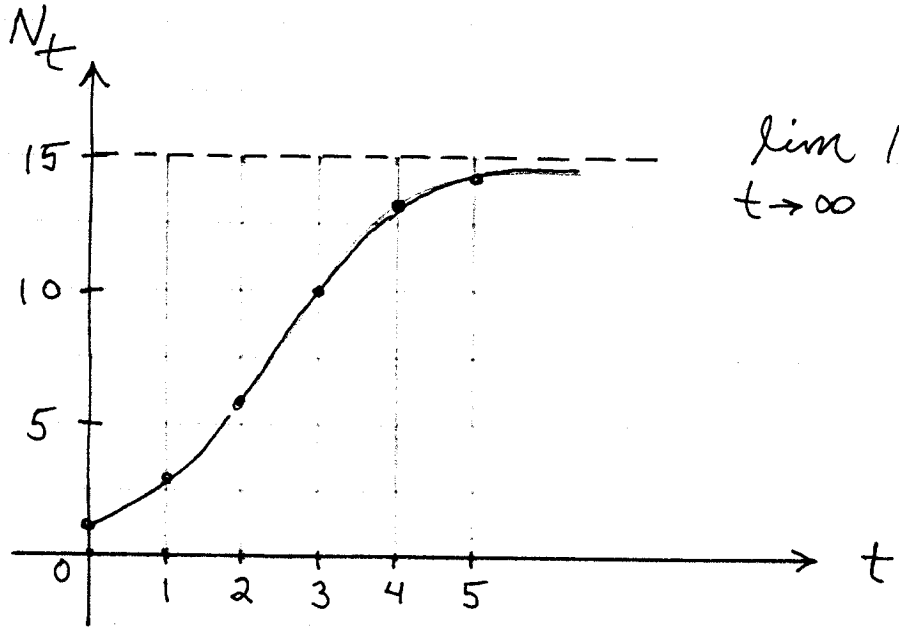
$$N_1 = \frac{45N_0}{15+2N_0} = \frac{45(1)}{15+2(1)} = \frac{45}{17} \approx 2.65\ ;$$

$$N_2 = \frac{45N_1}{15+2N_1} \approx \frac{45(2.65)}{15+2(2.65)} \approx 5.87\ ;$$

$$N_3 = \frac{45N_2}{15+2N_2} \approx \frac{45(5.87)}{15+2(5.87)} \approx 9.88\ ;$$

$$N_4 = \frac{45N_3}{15+2N_3} \approx \frac{45(9.88)}{15+2(9.88)} \approx 12.79 \;;$$

$$N_5 = \frac{45N_4}{15+2N_4} \approx \frac{45(12.79)}{15+2(12.79)} \approx 14.18$$



24.) $$N_{t+1} = \frac{4N_t}{1+\frac{4-1}{20}N_t} = \frac{4N_t}{1+\frac{3}{20}N_t} \cdot \frac{20}{20} = \frac{80N_t}{20+3N_t} \;;$$

t:	0	1	2	3	4	5
N_t:	10	16	18.82	19.69	19.92	19.98

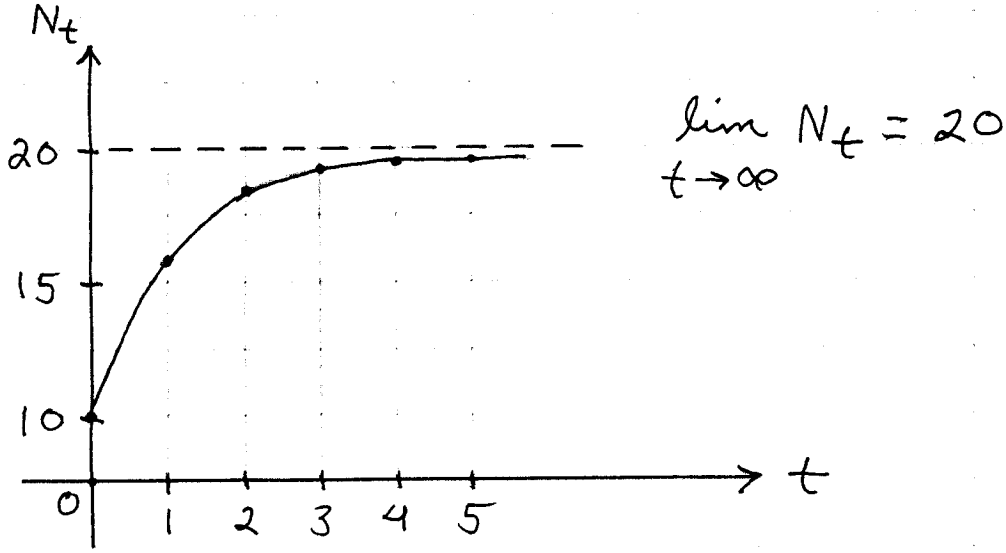
$$N_1 = \frac{80N_0}{20+3N_0} = \frac{80(10)}{20+3(10)} = 16 \;;$$

$$N_2 = \frac{80N_1}{20+3N_1} = \frac{80(16)}{20+3(16)} \approx 18.82 \;;$$

$$N_3 = \frac{80N_2}{20+3N_2} \approx \frac{80(18.82)}{20+3(18.82)} \approx 19.69 \;;$$

$$N_4 = \frac{80N_3}{20+3N_3} \approx \frac{80(19.69)}{20+3(19.69)} \approx 19.92 \;;$$

$$N_5 = \frac{80N_4}{20+3N_4} \approx \frac{80(19.92)}{20+3(19.92)} \approx 19.98$$



55.) t: 0 1 2 3 4 5 6 7 8 9

N_t: 1 1 2 3 5 8 13 21 34 55

t: 10 11 12 13 14 15 16 17 18 19 20

N_t: 89 144 233 377 610 987 1597 2584 4181 6765 10,946

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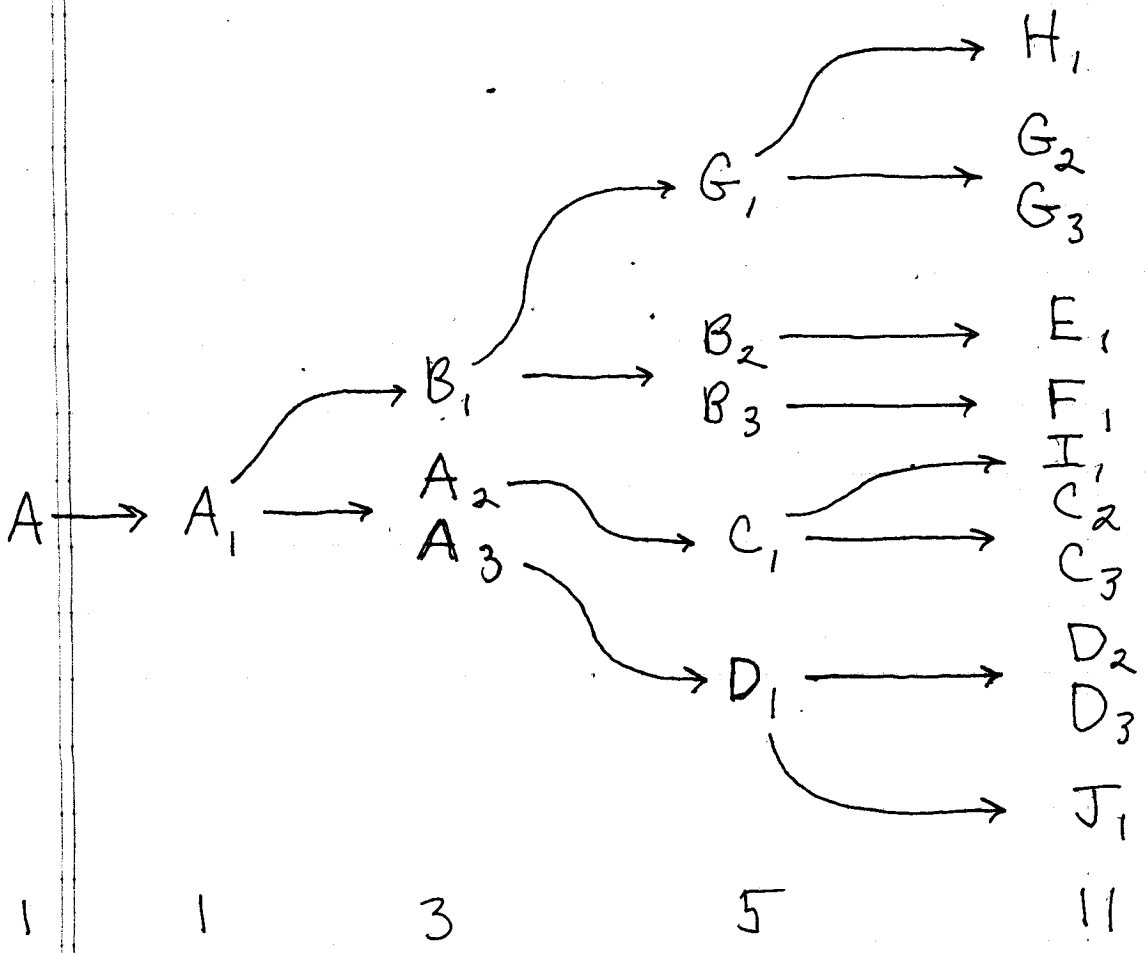
t:	2	3	4	5	6	7	8	9	10
N_t/N_{t-1}:	2	1.5	1.6667	1.6	1.625	1.6154	1.6190	1.6176	1.6182

t:	11	12	13	14	15	16	17
N_t/N_{t-1}:	1.6180	1.6181	1.61802	1.61804	1.618032	1.6180334	1.618033

t:	18	19	20
N_t/N_{t-1}:	1.6180341	1.6180339	1.6180340

57.) Let N_t : # of newborn rabbit pairs at time t months ; assume each pair produce 1 pair at age 1 month and 2 pair at age 2 months :



1 1 3 5 11

This is consistent with $N_{t+1} = N_t + 2N_{t-1}$:

1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365, 2731, 5461, 10923, ...

Conjecture : $\frac{N_t}{N_{t-1}} \to 2$ as $t \to \infty$