Section 3.2

3.) $f(x) = x^3 - 2x + 1$ at $x = 2$:

i.) $f(2) = 2^3 - 2(2) + 1 = 5$

ii.) $\lim_{x \to 2} f(x) = \lim_{x \to 2} (x^3 - 2x + 1) = 8 - 4 + 1 = 5$

iii.) $\lim_{x \to 2} f(x) = 5 = f(2)$,

so f is continuous at $x = 2$

5.) $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$ at $x = 2$:

i.) $f(2) = 3$

ii.) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$

$= \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = 3$

iii.) $\lim_{x \to 2} f(x) = 3 = f(2)$,

so f is continuous at $x = 2$.

7.) $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ a & \text{if } x = 3 \end{cases}$;

$\lim_{x \to 3} f(x) = \lim_{x \to 3} \dfrac{x^2 - 9}{x - 3} = \lim_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$

so choose $f(3) = a = 6$.

8.) $f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & \text{if } x \neq 1 \\ a & \text{if } x = 1 \end{cases}$;

i.) $f(1) = a$

ii.) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x \to 1} \dfrac{x^2+x-2}{x-1}$

"$\frac{0}{0}$" $\lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = 1+2 = 3$

iii.) Let $\boxed{a=3}$, then

$\lim\limits_{x\to 1} f(x) = 3 = f(1)$ and f is continuous at $x=1$.

9.) $f(x) = \begin{cases} \dfrac{1}{x-3} & \text{if } x \neq 3 \\ 0 & \text{if } x = 3 \end{cases}$ at $x = 3$:

i.) $f(3) = 0$

ii.) $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{1}{x-3} = \dfrac{1}{0} = \pm\infty$ (not finite)

so f is not continuous at $x=3$.

10.) $f(x) = \dfrac{1}{x^2-1}$; $y=1$ is cont. for all x-values (line) and $y = x^2-1$ is cont. for all x-values (parabola). Thus, $f(x) = \dfrac{1}{x^2-1}$ is continuous (quotient) for all x-values, except where

$x^2-1=(x-1)(x+1)=0$, i.e., except at $x=1$ and $x=-1$.

11.) $f(x)=\begin{cases}\dfrac{x^2-3x+2}{x-2} & \text{if } x\neq 1\\ 1 & \text{if } x=1\end{cases}$

<u>Check</u> $x=1$:

i.) $f(1)=1$

ii.) $\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1}\dfrac{x^2-3x+2}{x-2}=\dfrac{1-3+2}{1-2}=0$

iii.) $f(1)=1\neq 0=\lim\limits_{x\to 1} f(x)$

so f is <u>NOT continuous at $x=1$</u>;

in addition, $y=x^2-3x+2$ and $y=x-2$ are both continuous for all values of x (both are polynomials), so

$y=\dfrac{x^2-3x+2}{x-2}$ is continuous for all values of x (quotient), except where $x-2=0$, i.e., <u>except at $x=2$</u>.

12.) $f(x)=\begin{cases}x^2-1 & \text{if } x\leq 0\\ x & \text{if } x>0\end{cases}$ at $x=0$:

i.) $f(0)=0^2-1=0-1=-1$

ii.) $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+} x=0$ and

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}(x^2-1)=-1$, so

$\lim\limits_{x\to 0} f(x)$ DNE; thus, f is <u>not continuous at $x=0$</u>;

note that $y = x^2 - 1$ (parabola) is continuous for all $x < 0$ and $y = x$ (line) is continuous for all $x > 0$.

Math 17A
Kouba
Worksheet 1

1.) Use the three-step definition of continuity for each of the following problems.

a.) Let $f(x) = \begin{cases} x^2+3 , & \text{if } x \neq -1 \\ 2 , & \text{if } x = -1. \end{cases}$ Determine if f is continuous at $x = -1$.

b.) Let $g(x) = \begin{cases} x+1 , & \text{if } x \geq 0 \\ 2-x^2 , & \text{if } x < 0. \end{cases}$ Determine if g is continuous at $x = 0$.

c.) Let $f(x) = \begin{cases} x-2 , & \text{if } x > 1 \\ 0 , & \text{if } x = 1 \\ -x , & \text{if } x < 1. \end{cases}$ Determine if f is continuous at $x = 1$.

2.) Use approved shortcuts and facts from class to determine (with a brief explanation) the x-values for which each of the following functions is continuous.

a.) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ b.) $g(x) = \dfrac{\sin x}{x^2+4}$

c.) $f(x) = \dfrac{x+3}{x^2-4}$ d.) $g(x) = \cos(x^3 - x)$

3.) Determine constants A and B so that each of the following functions is continuous for all values of x. Start by drawing a"fake" graph. Then use limits.

a.) $f(x) = \begin{cases} \dfrac{x^2-7x+6}{x-6} , & \text{if } x \neq 6 \\ A , & \text{if } x = 6. \end{cases}$

b.) $f(x) = \begin{cases} A^2x - A , & \text{if } x \geq 1 \\ 2 , & \text{if } x < 1. \end{cases}$

c.) $f(x) = \begin{cases} \dfrac{A+x}{A+1} , & \text{if } x < 0 \\ Ax^3+3 , & \text{if } x \geq 0 \end{cases}$

d.) $f(x) = \begin{cases} 3 , & \text{if } x \leq 1 \\ Ax^2+B , & \text{if } 1 < x \leq 2 \\ 5 , & \text{if } x > 2 \end{cases}$

e.) $f(x) = \begin{cases} Ax - B , & \text{if } x \leq -1 \\ 2x+3A+B , & \text{if } -1 < x \leq 1 \\ 4 , & \text{if } x > 1 \end{cases}$

worksheet 1

1.) a.) $f(x) = \begin{cases} x^2+3 & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases}$; at $x = -1$:

i.) $f(-1) = 2$

ii.) $\lim_{x \to -1} f(x) = \lim_{x \to -1} (x^2+3) = 1+3 = 4$

iii.) $\lim_{x \to -1} f(x) = 4 \neq 2 = f(-1)$ so

f is NOT continuous at $x = -1$

b.) $g(x) = \begin{cases} x+1 & \text{if } x \geq 0 \\ 2-x^2 & \text{if } x < 0 \end{cases}$; at $x = 0$:

i.) $g(0) = 0+1 = 1$

ii.) $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x+1) = 1$;

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (2-x^2) = 2$; so

$\lim_{x \to 0} g(x)$ DNE ; thus

g is NOT continuous at $x = 0$

c.) $f(x) = \begin{cases} x-2 & \text{if } x > 1 \\ 0 & \text{if } x = 1 \\ -x & \text{if } x < 1 \end{cases}$; at $x = 1$:

i.) $f(1) = 0$

ii.) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x-2) = -1$;

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-x) = -1$; so

$\lim_{x \to 1} f(x) = -1$

iii.) $\lim_{x \to 1} f(x) = -1 \neq 0 = f(1)$, so

f is NOT continuous at $x = 1$.

2.) a.) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ is continuous for all values of x since it is a polynomial.

b.) $g(x) = \dfrac{\sin x}{x^2+4}$; $Y = \sin x$ (well known) and $Y = x^2 + 4$ (polynomial) are continuous for all values of x ; so the QUOTIENT $g(x) = \dfrac{\sin x}{x^2+4}$ is continuous for all values of x since $x^2 + 4 \neq 0$.

c.) $f(x) = \dfrac{x+3}{x^2-4}$; $Y = x+3$ and $Y = x^2 - 4$ (polynomials) are continuous for all values of x ; the QUOTIENT $f(x) = \dfrac{x+3}{x^2-4}$ is continuous for all values of x except where $x^2 - 4 = 0$, i.e., except when $x = \pm 2$.

d.) $g(x) = \cos(x^3 - x)$; $f(x) = \cos x$ (well known) and $k(x) = x^3 - x$ (polynomial) are continuous for all values of x ; so the COMPOSITION

$f(k(x)) = f(x^3 - x) = \cos(x^3 - x)$ is continuous for all values of x.

3.) a.) $f(x) = \begin{cases} \dfrac{x^2 - 7x + 6}{x - 6} & \text{if } x \neq 6 \\ A & \text{if } x = 6 \end{cases}$;

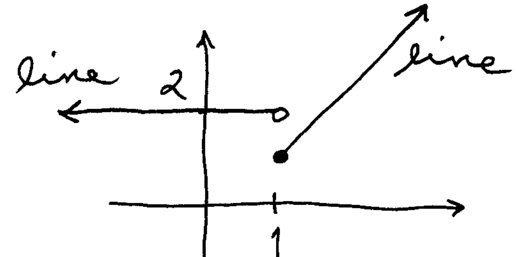
$\lim\limits_{x \to 6} f(x) = \lim\limits_{x \to 6} \dfrac{x^2 - 7x + 6}{x - 6} = \lim\limits_{x \to 6} \dfrac{\cancel{(x-6)}(x-1)}{\cancel{x-6}}$

$= 6 - 1 = 5$ and $f(6) = A$ so choose $A = 5$

b.) $f(x) = \begin{cases} A^2 x - A & \text{if } x \geq 1 \\ 2 & \text{if } x < 1 \end{cases}$;

Require that

$\lim\limits_{x \to 1^+} f(x) = 2 \rightarrow$

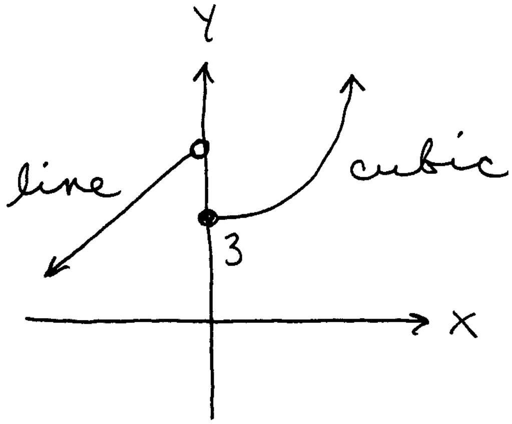


$\lim\limits_{x \to 1^+} (A^2 x - A) = 2 \rightarrow$

$A^2 - A = 2 \rightarrow A^2 - A - 2 = 0 \rightarrow$

$(A-2)(A+1) = 0 \rightarrow$ $A = 2$ or $A = -1$.

c.) $f(x) = \begin{cases} \dfrac{A + x}{A + 1} & \text{if } x < 0 \\ Ax^3 + 3 & \text{if } x \geq 0 \end{cases}$;



Require that

$\lim_{x \to 0^-} f(x) = 3 \rightarrow$

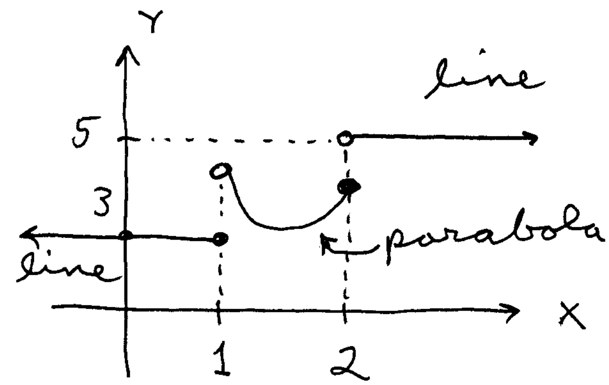
$\lim_{x \to 0^-} \frac{A+x}{A+1} = 3 \rightarrow$

$\frac{A+0}{A+1} = 3 \rightarrow A = 3A + 3 \rightarrow -3 = 2A \rightarrow$

$A = -3/2$

d.) $f(x) = \begin{cases} 3 & \text{if } x \le 1 \\ Ax^2 + B & \text{if } 1 < x \le 2 \\ 5 & \text{if } x > 2 \end{cases}$

Require <u>TWO</u> conditions :



i.) $\lim_{x \to 1^+} f(x) = 3 \rightarrow$

$\lim_{x \to 1^+} (Ax^2 + B) = 3 \rightarrow$

$\boxed{A + B = 3}$ AND

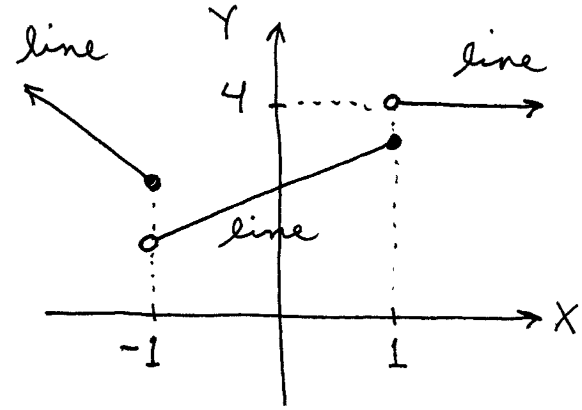
ii.) $\lim_{x \to 2^-} f(x) = 5 \rightarrow \lim_{x \to 2^-} (Ax^2 + B) = 5 \rightarrow$

$\boxed{4A + B = 5}$; solve system :

$B = 3 - A \rightarrow$ (SUB) $\rightarrow 4A + (3 - A) = 5 \rightarrow$

$3A = 2 \rightarrow$ $A = 2/3 \, , \, B = 7/3$

e.) $f(x) = \begin{cases} Ax - B & \text{if } x \le -1 \\ 2x + 3A + B & \text{if } -1 < x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$



Require TWO conditions :

i.) $\lim_{x \to 1^-} f(x) = 4 \longrightarrow$

$\lim_{x \to 1^-} (2x + 3A + B) = 4 \longrightarrow$

$2 + 3A + B = 4 \longrightarrow \boxed{3A + B = 2}$ AND

ii.) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) \longrightarrow$

$\lim_{x \to -1^-} (Ax - B) = \lim_{x \to -1^+} (2x + 3A + B) \longrightarrow$

$-A - B = -2 + 3A + B \longrightarrow 2 = 4A + 2B \longrightarrow$

$\boxed{2A + B = 1}$; solve system :

$3A + B = 2 \longrightarrow B = 2 - 3A \longrightarrow$ (SUB) $\longrightarrow$

$2A + (2 - 3A) = 1 \longrightarrow -A = -1 \longrightarrow$

$A = 1 , \quad B = -1$.