

Section 3.3

$$\begin{aligned}
 1.) \quad \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 5}{x^4 - 2x + 1} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 5}{x^4 - 2x + 1} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{2}{x^2} - \frac{3}{x} + \frac{5}{x^4}}{1 - \frac{2}{x^3} + \frac{1}{x^4}} = \frac{0 - 0 + 0}{1 - 0 + 0} = \frac{0}{1} = 0
 \end{aligned}$$

$$\begin{aligned}
 2.) \quad \lim_{x \rightarrow \infty} \frac{x^2 + 3}{5x^2 - 2x + 1} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{x^2 + 3}{5x^2 - 2x + 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
 &= \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x^2}}{5 - \frac{2}{x} + \frac{1}{x^2}} = \frac{1 + 0}{5 - 0 + 0} = \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 3.) \quad \lim_{x \rightarrow \infty} \frac{x^3 + 3}{x - 2} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{x^3 + 3}{x - 2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
 &= \lim_{x \rightarrow \infty} \frac{x^2 + \frac{3}{x}}{1 - \frac{2}{x}} = \frac{\infty + 0}{1 - 0} = \infty
 \end{aligned}$$

$$\begin{aligned}
 8.) \quad \lim_{x \rightarrow -\infty} \frac{3 - x^2}{1 - 2x^2} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow -\infty} \frac{3 - x^2}{1 - 2x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
 &= \lim_{x \rightarrow -\infty} \frac{\frac{3}{x^2} - 1}{\frac{1}{x^2} - 2} = \frac{0 - 1}{0 - 2} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 9.) \quad \lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 1}{4 - x} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 1}{4 - x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{x - 3 + \frac{1}{x}}{\frac{4}{x} - 1} = \frac{-\infty - 3 + 0}{0 - 1} = \frac{-\infty}{-1} = \infty
 \end{aligned}$$

$$\begin{aligned}
 10.) \quad \lim_{x \rightarrow -\infty} \frac{1 - x^3}{2 + x} &\stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow -\infty} \frac{1 - x^3}{2 + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{\frac{1}{x} - x^2}{\frac{2}{x} + 1} = \frac{0 - \infty}{0 + 1} = -\infty
 \end{aligned}$$

$$13.) \quad \lim_{x \rightarrow \infty} \frac{4}{1 + e^{-2x}} = \lim_{x \rightarrow \infty} \frac{4}{1 + \frac{1}{e^{2x}}} = \frac{4}{1 + \frac{1}{\infty}} = \frac{4}{1 + 0} = 4$$

$$14.) \lim_{x \rightarrow \infty} \frac{e^{-x}}{1-e^{-x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{e^x}}{1-\frac{1}{e^x}} = \frac{\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0}{1-0} = 0$$

$$15.) \lim_{x \rightarrow \infty} \frac{2e^x}{e^x+3} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{2e^x}{e^x+3} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{1+\frac{3}{e^x}} = \frac{2}{1+\frac{3}{\infty}} = \frac{2}{1+0} = 2$$

$$16.) \lim_{x \rightarrow \infty} \frac{e^x}{2-e^x} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2-e^x} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\frac{2}{e^x}-1} = \frac{1}{\frac{2}{\infty}-1} = \frac{1}{0-1} = -1$$

$$18.) \lim_{x \rightarrow \infty} e^{-\ln x} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$19.) \lim_{x \rightarrow \infty} \frac{3e^{2x}}{2e^{2x}-e^x} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{3e^{2x}}{2e^{2x}-e^x} \cdot \frac{\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}}$$

$$= \lim_{x \rightarrow \infty} \frac{3}{2-\frac{1}{e^x}} = \frac{3}{2-\frac{1}{\infty}} = \frac{3}{2-0} = \frac{3}{2}$$

$$20.) \lim_{x \rightarrow \infty} \frac{3e^{2x}}{2e^{2x}-e^{3x}} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{3e^{2x}}{2e^{2x}-e^{3x}} \cdot \frac{\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}}$$

$$= \lim_{x \rightarrow \infty} \frac{3}{2-e^x} = \frac{3}{2-\infty} = \frac{3}{-\infty} = 0$$

$$21.) \lim_{x \rightarrow \infty} \frac{3}{2+e^{-x}} = \lim_{x \rightarrow \infty} \frac{3}{2+\frac{1}{e^x}} = \frac{3}{2+\frac{1}{\infty}}$$

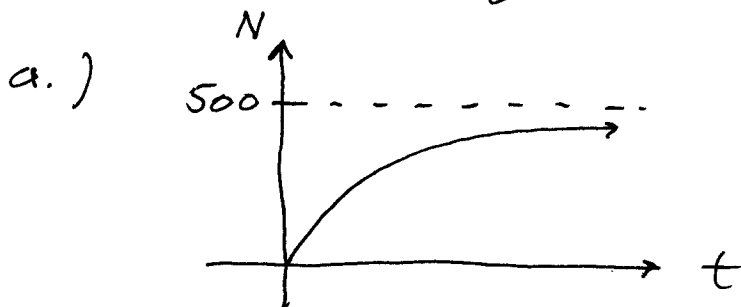
$$= \frac{3}{2+0} = \frac{3}{2}$$

$$22.) \lim_{x \rightarrow -\infty} \frac{4}{1+e^{-x}} = \frac{4}{1+e^{\infty}} = \frac{4}{1+\infty} = \frac{4}{\infty} = 0$$

$$23.) \lim_{x \rightarrow -\infty} \frac{e^x}{1+x} = \frac{e^{-\infty}}{1-\infty} = \frac{\frac{1}{e^{\infty}}}{-\infty} = \frac{0}{-\infty} = 0$$

$$24.) \lim_{x \rightarrow \infty} \frac{2}{e^x(1+x)} = \frac{2}{\infty \cdot \infty} = \frac{2}{\infty} = 0$$

$$27.) N(t) = \frac{500t}{3+t} \text{ for } t \geq 0$$

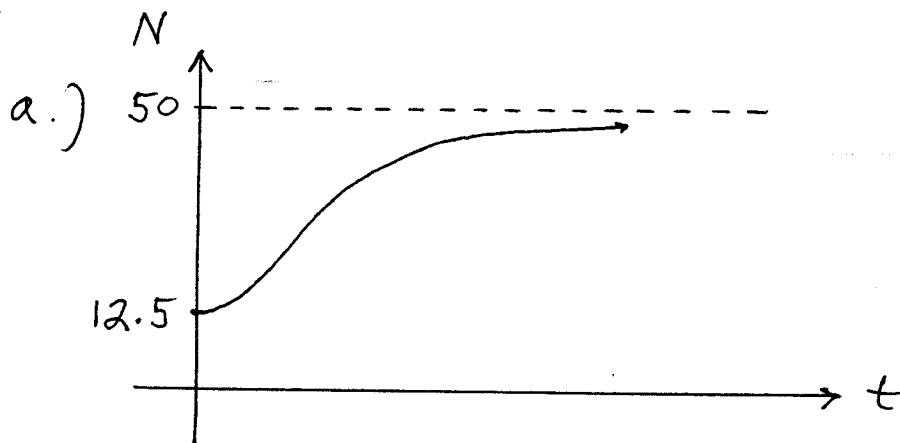


$$b.) \lim_{t \rightarrow \infty} \frac{500t}{3+t} \stackrel{\text{"\infty/\infty"}}{=} \lim_{t \rightarrow \infty} \frac{500t \cdot \frac{1}{t}}{3+t \cdot \frac{1}{t}}$$

$$= \lim_{t \rightarrow \infty} \frac{500}{\frac{3}{t} + 1} = \frac{500}{0+1} = 500$$

c.) If $t=3$, then $N = \frac{500(3)}{3+3} = 250$

$$29.) \quad N(t) = \frac{50}{1+3e^{-t}} \quad \text{for } t \geq 0$$



$$\begin{aligned} b.) \quad \lim_{t \rightarrow \infty} \frac{50}{1+3e^{-t}} &= \lim_{t \rightarrow \infty} \frac{50}{1+3 \cdot \frac{1}{e^t}} \\ &= \frac{50}{1+3 \cdot \left(\frac{1}{\infty}\right)} = \frac{50}{1+3(0)} = 50 \end{aligned}$$