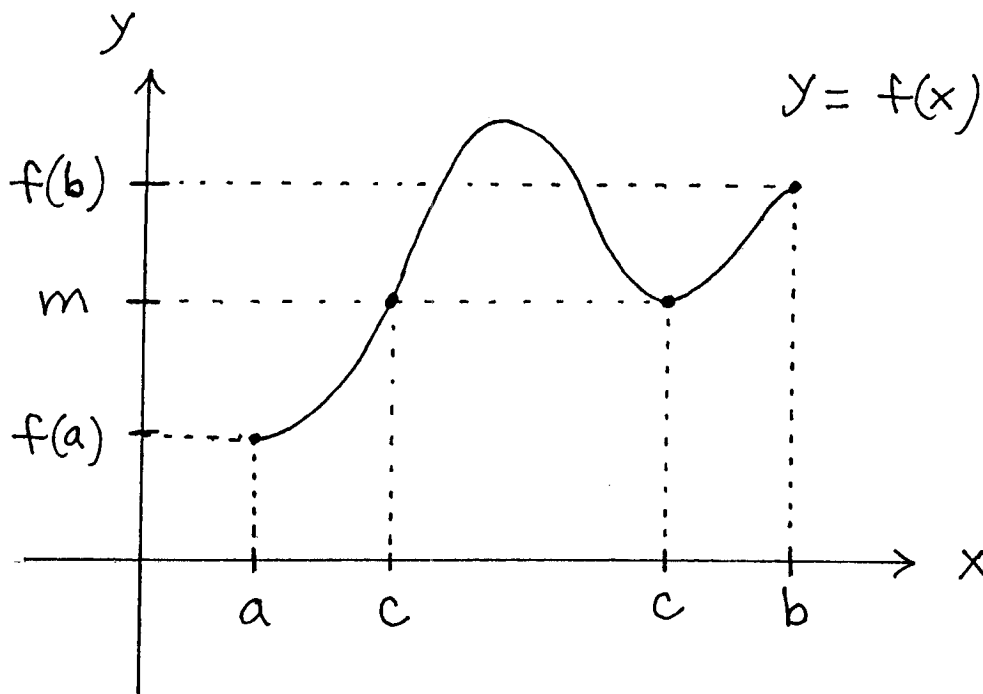


Math 17A
Kouba

Intermediate Value Theorem (IMVT) : Let f be a continuous function on the closed interval $[a, b]$. Let m be any number between $f(a)$ and $f(b)$. Then there is at least one number c in $[a, b]$ which satisfies

$$f(c) = m .$$



When applying the IMVT to a problem, the following five steps must be clearly established:

1. Define a function f .
2. Define a number m .
3. Establish that f is continuous.
4. Choose an interval $[a, b]$.
5. Indicate that m is between $f(a)$ and $f(b)$.

Once these five steps have been established, the conclusion of the IMVT can be invoked.

Math 17A

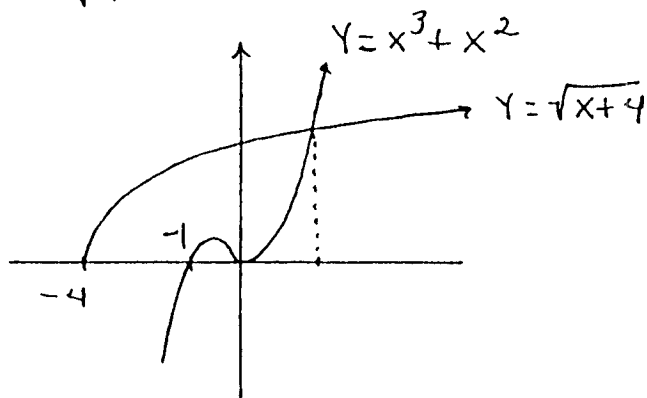
Kouba

an Example Using the Intermediate Value Theorem

IMVT

Ex: Determine if $x^3 + x^2 = \sqrt{x+4}$ is solvable.

Begin by sketching $y = x^3 + x^2 = x^2(x+1)$ and $y = \sqrt{x+4}$:



It appears that there is a solution. Now we must prove its existence. Since

$$x^3 + x^2 = \sqrt{x+4} \text{ then}$$

$$x^3 + x^2 - \sqrt{x+4} = 0 \text{ so}$$

let $f(x) = x^3 + x^2 - \sqrt{x+4}$ and let $m = 0$.

By trial and error $f(0) = -2$ and $f(5) = 147$, and $m = 0$ is between $f(0)$ and $f(5)$.

Thus, since f is a continuous function (It is sum and difference of continuous functions.) on a closed interval $[0, 5]$, it follows from the IMVT that there is at least one number c in $[0, 5]$ so that

$$f(c) = m, \text{ i.e.,}$$

$$c^3 + c^2 - \sqrt{c+4} = 0, \text{ i.e.,}$$

$$c^3 + c^2 = \sqrt{c+4}.$$

Thus, we have proven that the original equation is solvable.