

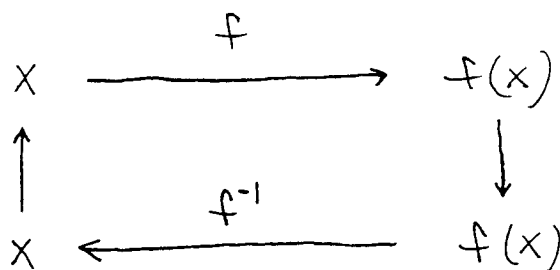
Math 17A

Kouba

Inverse Function Summary

I. If $f(x)$ is a one-to-one function, then $f(x)$ has an inverse function, $f^{-1}(x)$, which satisfies:

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x.$$



- The domain of $f^{-1}(x)$ is the range of $f(x)$.
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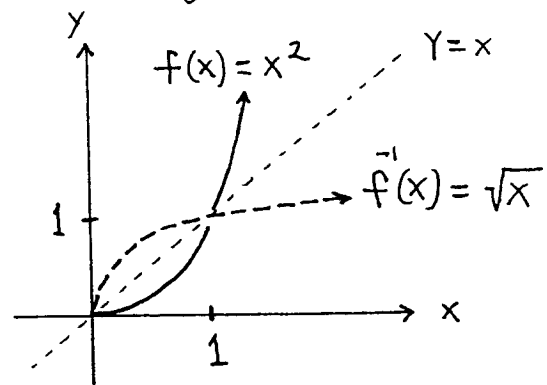
Ex: Let $f(x) = x^2$ for $x \geq 0$,
then

$$f^{-1}(x) = \sqrt{x};$$

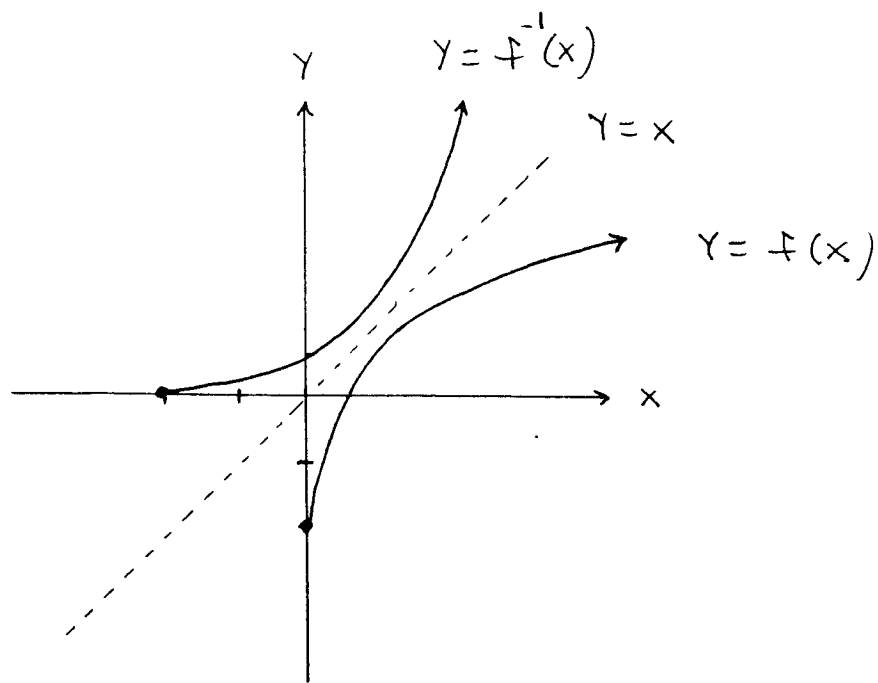
$$f(3) = 9 \quad \text{and} \quad f^{-1}(9) = 3$$

so that

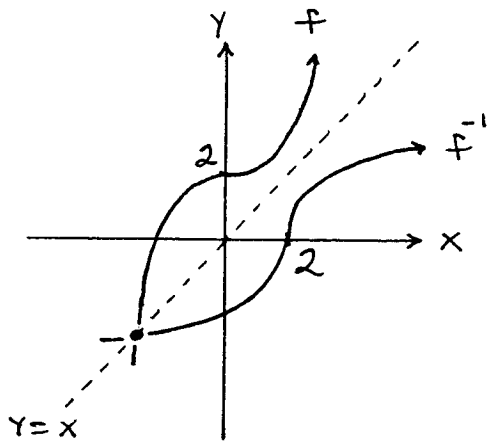
$$f(f^{-1}(9)) = f(3) = 9 \quad \text{and} \quad f^{-1}(f(3)) = f^{-1}(9) = 3.$$



II. If $f(x)$ and $f^{-1}(x)$ are graphed on the same axes, they are symmetric with respect to the line $Y = X$.



Ex: Graph $f(x) = x^5 + 2$ to verify that it is one-to-one, and find its inverse:



$$f(f^{-1}(x)) = x \text{ so that}$$

$$(f^{-1}(x))^5 + 2 = x \rightarrow$$

$$(f^{-1}(x))^5 = x - 2 \rightarrow$$

$$f^{-1}(x) = (x - 2)^{1/5}$$

Extra Problems: Find $f^{-1}(x)$ for

1.) $f(x) = 7 - \frac{2}{x}$, where $x \neq 0$

2.) $f(x) = \frac{x+3}{x-1}$, where $x \neq 1$

3.) $f(x) = x^2 - 6x$, where $x \leq 3$