

Math 17A

Kouba

Finding % Errors Using a Differential

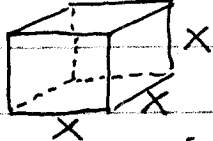
Let $y = f(x)$ be a differentiable function;

and Δx : change (error) in x ,

Δf : change (error) in f ; then

$\frac{|\Delta x|}{x}$: absolute relative (%) error in x ,

$\frac{|\Delta f|}{f}$: absolute relative (%) error in f ;

Example:  The edge of a cube is measured with possible % error of at most 2%. Use a differential to estimate the maximum % error in computing the cube's volume:

$V = x^3 \xrightarrow{D} V' = 3x^2$, and $\frac{|\Delta x|}{x} \leq 2\%$;
estimate the value of $\frac{|\Delta V|}{V}$. Then

$$\begin{aligned}\frac{|\Delta V|}{V} &\approx \frac{|dV|}{V} = \frac{|V' \cdot \Delta x|}{V} = \frac{|3x^2 \cdot \Delta x|}{x^3} \\ &= \frac{3x^2 \cdot |\Delta x|}{x^3} = 3 \cdot \frac{|\Delta x|}{x} \leq 3 \cdot (2\%) \\ &= 6\%, \text{ i.e., } \frac{|\Delta V|}{V} \leq 6\%.\end{aligned}$$