

Math 17A
Kouba
Derivative of $\sin x$

Facts: 1. $\lim_{z \rightarrow 0} \frac{\sin z}{z} = 1$

2. $\lim_{z \rightarrow 0} \frac{\cos z - 1}{z} = 0$

3. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

If $f(x) = \sin x$, then $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$

Fact 3 \Downarrow
 $= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cdot (\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h \rightarrow 0} \left\{ \sin x \cdot \left(\frac{\cos h - 1}{h} \right) + \cos x \cdot \left(\frac{\sin h}{h} \right) \right\}$$

Facts 1+2 \Downarrow

$$= \sin x \cdot (0) + \cos x \cdot (1)$$

$$= \cos x, \quad \text{i.e.,}$$

$$D(\sin x) = \cos x.$$