

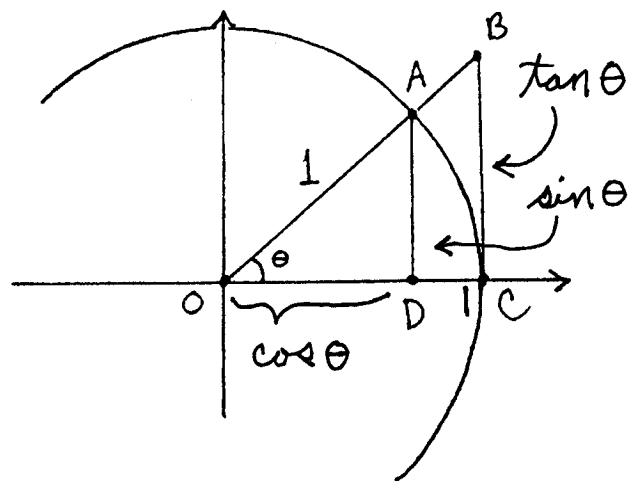
# Math 17A

Kouba

## A Useful Limit

Problem: Determine  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$ , where  $\theta$  is given in radians.

Start by assuming that  $\theta > 0$  is in the first quadrant, and assume the circle has radius 1.



It follows that

$$\text{Area } \triangle OAD < \text{Area } \triangle OAC < \text{Area } \triangle OBC \longrightarrow$$

$$\frac{1}{2}(\cos \theta)(\sin \theta) < \frac{\theta}{2\pi} \cdot \pi(1)^2 < \frac{1}{2}(1)(\tan \theta) \longrightarrow$$

$$(\cos \theta)(\sin \theta) < \theta < \frac{\sin \theta}{\cos \theta} \longrightarrow$$

$$\cos \theta < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta} \longrightarrow$$

$$\frac{1}{\cos \theta} > \frac{\sin \theta}{\theta} > \cos \theta \longrightarrow$$

$$\lim_{\theta \rightarrow 0} \frac{1}{\cos \theta} \geq \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \geq \lim_{\theta \rightarrow 0} \cos \theta \longrightarrow$$

$$\frac{1}{1} \geq \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \geq 1 \quad \text{(Squeeze Principle)}$$

$$\boxed{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1}$$

(similarly for  $\theta < 0$ ).