Math 17A (Fall 2016)
Kouba
Exam 2

Please PRINT your name here : KEY

Your Exam ID Number ________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 7 pages, including the cover page.

6. Put units on answers where units are appropriate.

7. You will be graded on proper use of limit and derivative notation.

8. You have until 11:58 a.m. sharp to finish the exam.

1.) (7 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = 4x^{2/3} + (1/2)x^{-4} - 1000^{50}$ $\xrightarrow{D}$

$$y' = 4 \cdot \frac{2}{3} x^{-1/3} + \frac{1}{2}(-4)x^{-5} - 0$$

b.) $f(x) = \dfrac{2x - x^2}{x^5 + 3x^2 - 1}$ $\xrightarrow{D}$

$$f'(x) = \frac{(x^5 + 3x^2 - 1)(2 - 2x) - (2x - x^2)(5x^4 + 6x)}{(x^5 + 3x^2 - 1)^2}$$

c.) $f(x) = \dfrac{x^3 + 7}{(x^2 - x)^4}$ $\xrightarrow{D}$

$$f'(x) = \frac{(x^2 - x)^4 \cdot 3x^2 - (x^3 + 7) \cdot 4(x^2 - x)^3 (2x - 1)}{(x^2 - x)^8}$$

2.) (5 pts. each) Let $f(x) = (2/3)x^{3/2} - 4\sqrt{x}$.

a.) Solve $f'(x) = 0$ for x .

$$\xrightarrow{D} f'(x) = \frac{2}{3} \cdot \frac{3}{2} x^{1/2} - 4 \cdot \frac{1}{2} x^{-1/2} = \frac{\sqrt{x}}{1} - \frac{2}{\sqrt{x}}$$

$$= \frac{x - 2}{\sqrt{x}} = 0 \rightarrow x - 2 = 0 \rightarrow x = 2$$

b.) Solve $f''(x) = 0$ for x .

$$\xrightarrow{D} f''(x) = \frac{\sqrt{x}(1) - (x-2) \cdot \frac{1}{2} x^{-1/2}}{x}$$

$$= \frac{\frac{\sqrt{x}}{1} - \frac{x-2}{2\sqrt{x}}}{\frac{x}{1}} = \frac{2x - x + 2}{2\sqrt{x}} \cdot \frac{1}{x} = \frac{x+2}{2x^{3/2}} = 0 \rightarrow$$

$x + 2 = 0 \rightarrow x = -2$ Impossible so no solution

3.) (8 pts.) Use the Intermediate Value Theorem to prove that the equation $x^3 = x + 3$ is solvable. This is a writing exercise. You will be scored on proper writing style and mathematical correctness.

$x^3 = x+3 \rightarrow x^3 - x - 3 = 0$ so let $\boxed{m=0}$ and $\boxed{f(x) = x^3 - x - 3}$, where f is continuous for all x-values (polynomial); note that $f(0) = -3$ and $f(2) = 3$ and $m=0$ is $\boxed{\text{between}}$ $f(0)$ and $f(2)$, so choose interval $\boxed{[0,2]}$. Thus, by IMVT there is at least one x-value c, $0 \le c \le 2$, so that $f(c) = m$, i.e., $c^3 - c - 3 = 0$ and the equation is solvable.

4.) (8 pts.) The following table of values uses the Bisection Method to estimate the solution to the equation $f(x) = 0$, where $f(x) = x^3 - x^2 - 1$. Fill in the missing 7 numbers in the table.

a	b	(1/2)(a+b)	f(a)	f(b)	f((1/2)(a+b))
0	2	1	-1	3	-1
1	2	1.5	-1	3	0.125
1	1.5	1.25	-1	0.125	-0.609375

5.) (9 pts.) Assume that y is a function of x and $x^3 + y^3 = xy + 8$. Determine an equation of the line tangent to this graph at the point $x = 0$.

$X = 0 \longrightarrow 0 + Y^3 = 0 + 8 \rightarrow Y = 2$ and

$\xrightarrow{D} 3X^2 + 3Y^2Y' = XY' + (1)Y$

$\rightarrow 3Y^2Y' - XY' = Y - 3X^2$

$\rightarrow (3Y^2 - X)Y' = Y - 3X^2$

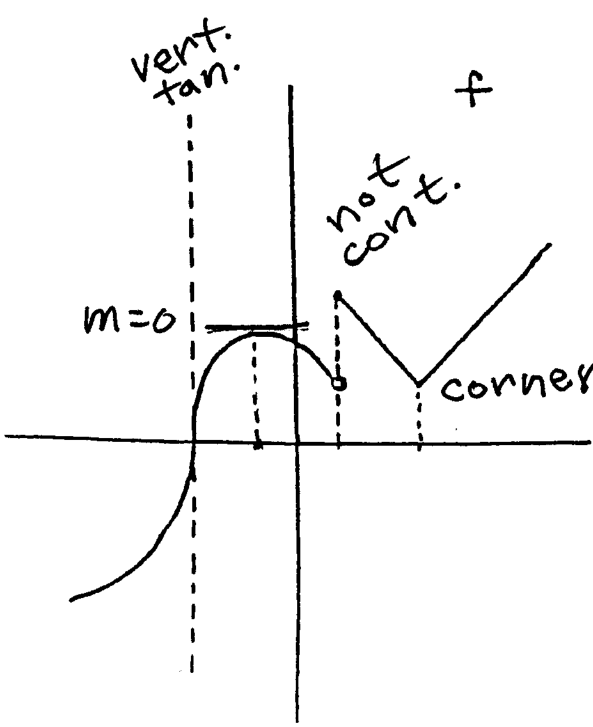
$\rightarrow Y' = \dfrac{Y - 3X^2}{3Y^2 - X}$; $X = 0, Y = 2 \rightarrow$

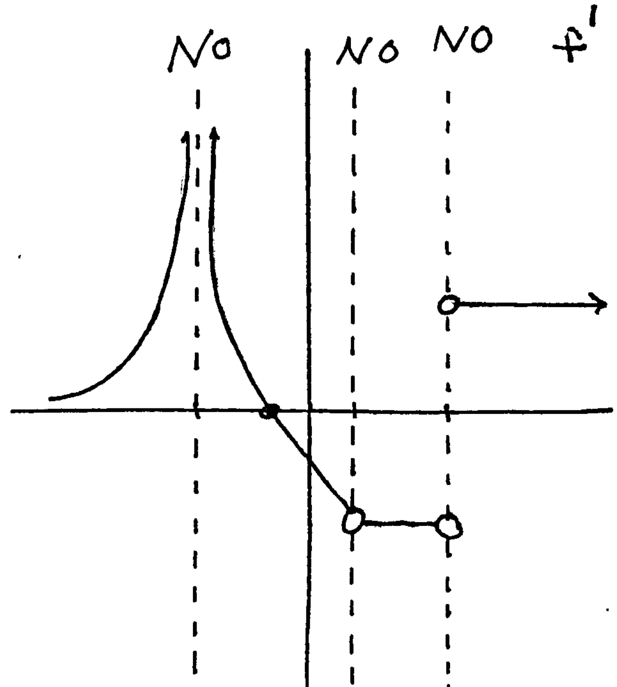
$m = Y' = \dfrac{2 - 3(0)^2}{3(2)^2 - 0} = \dfrac{2}{12} = \dfrac{1}{6}$ so

tangent line is

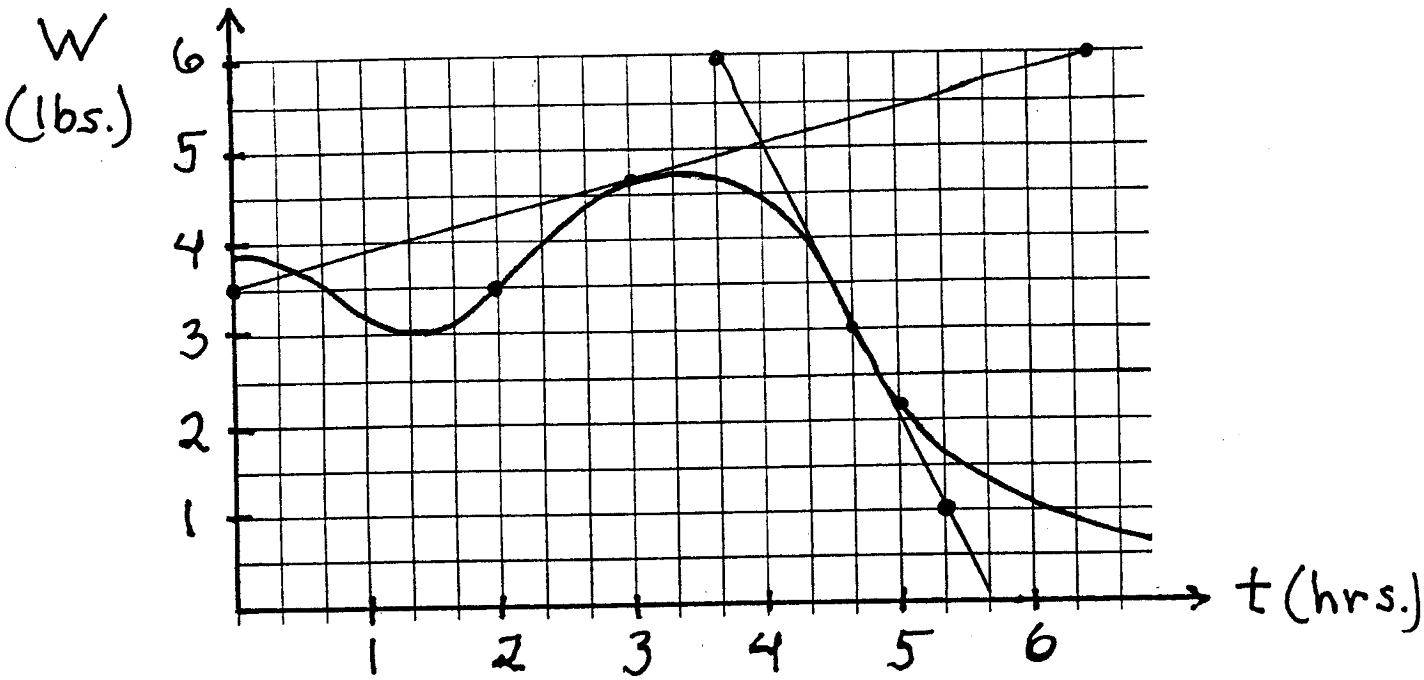
$Y - 2 = \frac{1}{6}(X - 0)$ or $Y = \frac{1}{6}X + 2$

6.) (8 pts.) Sketch a graph of the derivative f' using the given graph of f.





7.) The given graph represents the weight W (lbs.) of a growing (freezing) and shrinking (melting) block of ice from $t = 0$ hours to $t = 6$ hours.



a.) (4 pts.) Estimate the block's average rate of weight change for $t = 2$ to $t = 5$ hours.

$$ARC = \frac{2.2 - 3.5}{5 - 2} \approx -0.433 \text{ lbs./hr.}$$

b.) (4 pts.) Estimate the block's instantaneous rate of weight change when $t = 3$ hours.

$$IRC = \frac{6 - 3.5}{6.4 - 0} \approx 0.39 \text{ lbs./hr.}$$

c.) (4 pts.) Estimate the specific time t at which the block is shrinking most rapidly, and estimate the value of this rate of weight change.

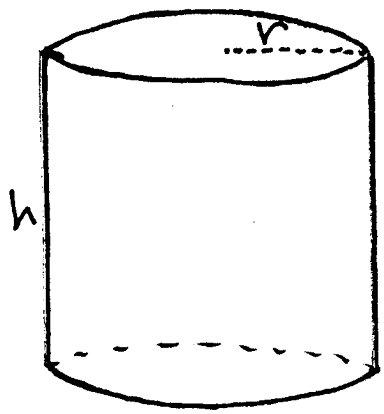
$t = 4.67$ hrs. and

$$IRC = \frac{-5}{1\frac{2}{3}} = -3 \text{ lbs./hr.}$$

8.) (8 pts.) Consider a closed right circular cylinder of radius r and height h. If the radius is increasing at the rate of 5 cm./sec. and the height is decreasing at the rate of 2 cm./sec. At what rate is the total surface area of the cylinder changing when $r = 3$ cm. and $h = 4$ cm. ?

Given $\frac{dr}{dt} = 5$ cm./sec., $\frac{dh}{dt} = -2$ cm./sec.,

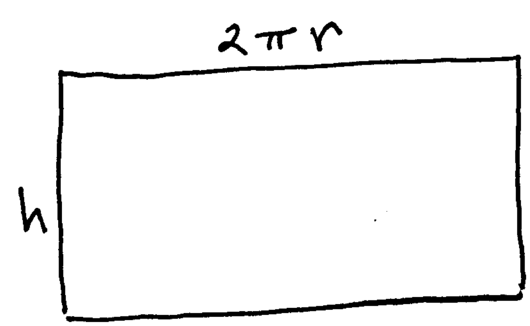
Find $\frac{dS}{dt}$ when $r = 3$ cm., $h = 4$ cm. :



$$S = 2\pi r^2 + 2\pi r h \xrightarrow{D}$$

$$\frac{dS}{dt} = 4\pi r \cdot \frac{dr}{dt} + 2\pi r \cdot \frac{dh}{dt} + 2\pi \frac{dr}{dt} \cdot h$$

$$= 4\pi(3)(5) + 2\pi(3)(-2) + 2\pi(5)(4)$$

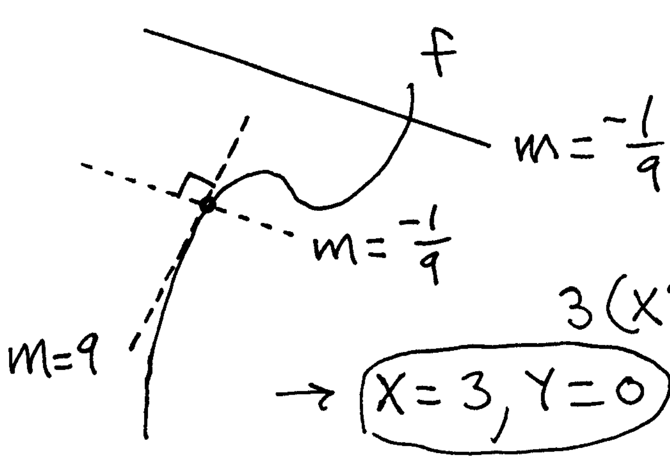


$$= 60\pi - 12\pi + 40\pi$$

$$= 88\pi \text{ cm.}^2/\text{sec}.$$

9.) (8 pts.) Find all points (x, y) on the graph of $f(x) = x^3 - 3x^2$ with tangent lines perpendicular to the graph of $x + 9y = 18$.

$X + 9Y = 18 \rightarrow 9Y = -X + 18 \rightarrow Y = \frac{-1}{9}X + 2$ has SLOPE $= \frac{-1}{9}$;



$$f'(X) = \boxed{3X^2 - 6X = 9} \rightarrow$$

$$3X^2 - 6X - 9 = 0 \rightarrow$$

$$3(X^2 - 2X - 3) = 0 \rightarrow 3(X-3)(X+1) = 0$$

$$\rightarrow X = 3, Y = 0 \quad \text{or} \quad X = -1, Y = -4$$

1
1 1
1 2 1
1 3 3 1

10.) (8 pts.) Use $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x) = x^3 - x^2 + 5$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - (x+h)^2 + \cancel{5} - (x^3 - x^2 + \cancel{5})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^2} - 2hx - h^2 \cancel{- x^3} \cancel{+ x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3hx + h^2 - 2x - h)}{\cancel{h}}$$

$$= 3x^2 + 0 + 0 - 2x - 0$$

$$= 3x^2 - 2x$$

The following EXTRA CREDIT PROBLEM is worth 8 points. This problem is OPTIONAL.

1.) Assume that constants a and b satisfy $1 < a < b$. Prove that the following equation is solvable:

$$\frac{a}{x} + x = \frac{1}{x-b}$$

(mult. by $x(x-b)$) $\rightarrow$

$a(x-b) + x^2(x-b) = x \rightarrow a(x-b) + x^2(x-b) - x = 0$

so let $f(x) = a(x-b) + x^2(x-b) - x$ and let $m = 0$;

f is cont. (polynomial); $f(0) = -ab < 0$ and

$$f(a+b) = a^2 + (a+b)^2 a - (a+b)$$

$$= \underbrace{\{a^2 - a\}}_{(+)} + \underbrace{\{(a+b)^2 a - b\}}_{(+)} \quad (\text{since } 1 < a < b)$$

> 0 ; $m = 0$ is between $f(0)$ and $f(a+b)$

so choose interval $[0, a+b]$; thus by IMVT there is # c, $0 < c < a+b$ so that $f(c) = 0$ and equation is solvable.