Math 17B Kouba

a Biological Example of Unstable Equilibrium

Allee Effect (Warder Allee, American Zoologist, 1885–1955) – The theory states that for very small populations, the reproduction and survival rates increase with increased population density, and the population will shrink and ultimately go to extinction if it falls below a certain threshold. This contrasts with large populations where increased population density often reduces the growth rate.

The following growth rate illustrates the Allee Effect.

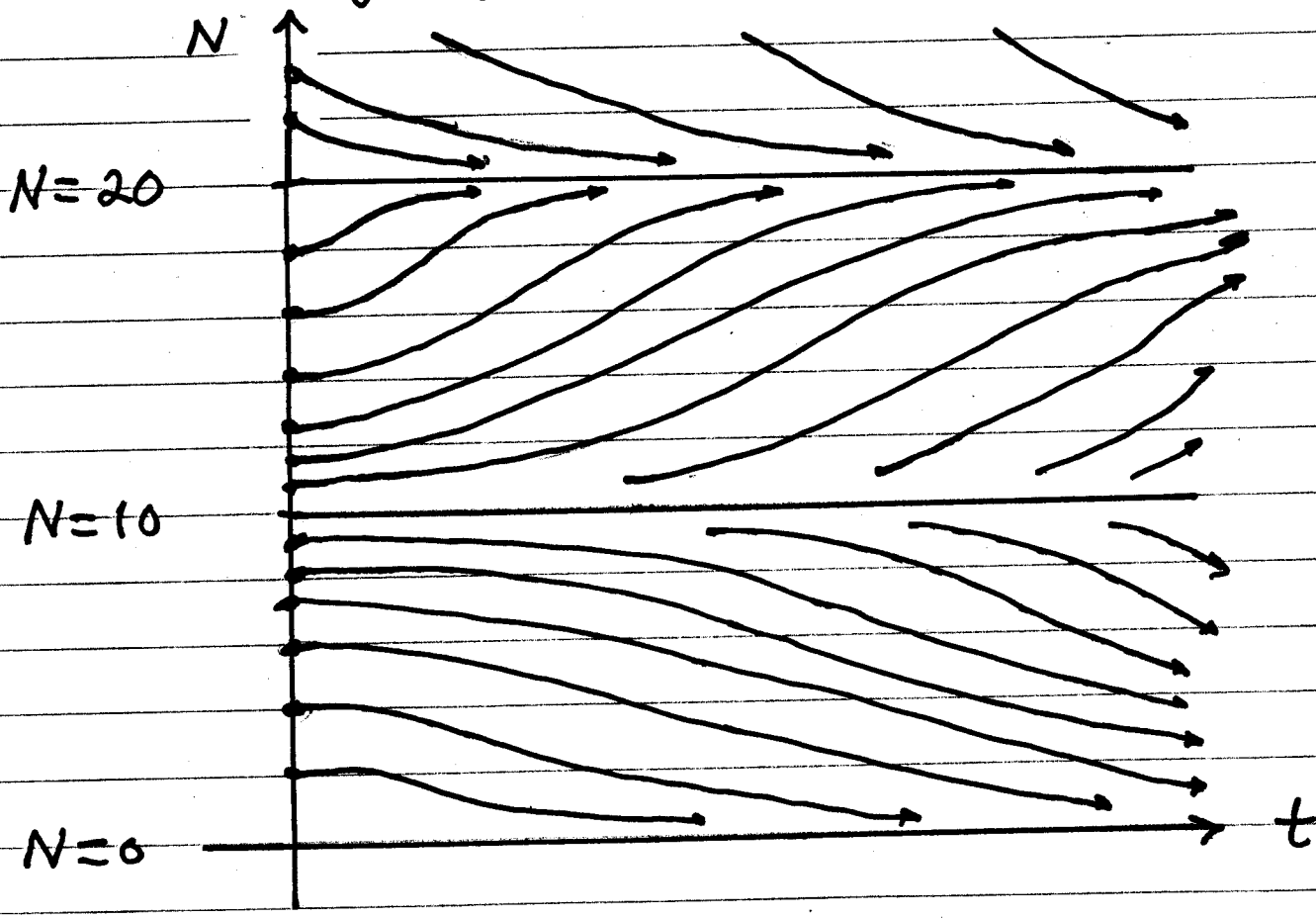
Example: $\frac{dN}{dt} = \underbrace{N(N-10)\left(1-\frac{N}{20}\right)}_{g(N)}$;

$g(N)=0 \longrightarrow N=0,\ N=10,\ N=20$ are equilibria; using the triple product rule it can be shown that

$$g'(N) = \frac{-3}{20}N^2 + 3N - 10$$

Using the eigenvalue method we get

$N=0$: $g'(0) = -10 < 0$ (Stable)
$N=10$: $g'(10) = 5 > 0$ (Unstable)
$N=20$: $g'(20) = -10 < 0$ (Stable)



The graphs suggest that if the initial population $N(0)$ is below 10, the population will eventually go extinct. If $N(0)$ is above 10, the population remains viable.