

Math 17B

Kouba

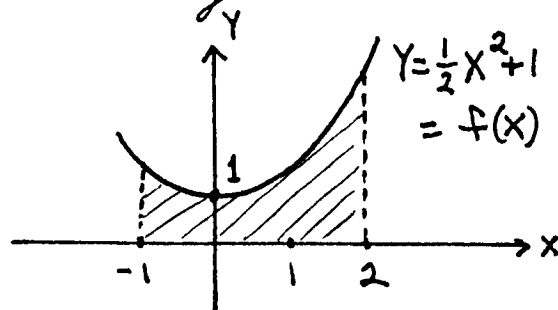
Definite Integral

Example: Use equal subintervals and the limit definition for a definite integral to evaluate $\int_{-1}^2 \left(\frac{1}{2}x^2 + 1\right) dx$.

Divide the interval $[-1, 2]$ into n equal parts, each of length $\frac{3}{n}$. Let the

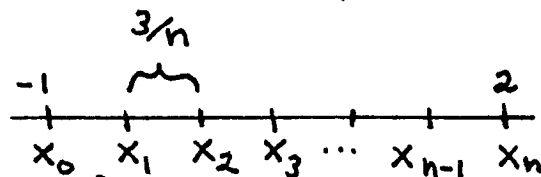
sampling points be the right-hand endpoints of the subintervals:

Then, by limit definition



$$\int_{-1}^2 \left(\frac{1}{2}x^2 + 1\right) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) (x_i - x_{i-1})$$

$x_i = -1 + \frac{3}{n}i$



$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{3}{n}i\right) \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{1}{2} \left(-1 + \frac{3}{n}i\right)^2 + 1 \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{9}{2n^2} i^2 - \frac{3}{n} i + \frac{3}{2} \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{27}{2n^3} i^2 - \frac{9}{n^2} i + \frac{9}{2n} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{27}{2n^3} \left(\sum_{i=1}^n i^2 \right) - \frac{9}{n^2} \left(\sum_{i=1}^n i \right) + \frac{9}{2n} \left(\sum_{i=1}^n 1 \right) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{27}{2n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{9}{n^2} \cdot \frac{n(n+1)}{2} + \frac{9}{2n} \cdot n \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{9}{4} \cdot \frac{2n^2 + 3n + 1}{n^2} - \frac{9}{2} \cdot \frac{n+1}{n} + \frac{9}{2} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{9}{4} \cdot \left(2 + \frac{3}{n} + \frac{1}{n^2} \right) - \frac{9}{2} \cdot \left(1 + \frac{1}{n} \right) + \frac{9}{2} \right]$$

$$= \frac{9}{4} \cdot 2 - \frac{9}{2} \cdot 1 + \frac{9}{2}$$

$$= \frac{9}{2}$$