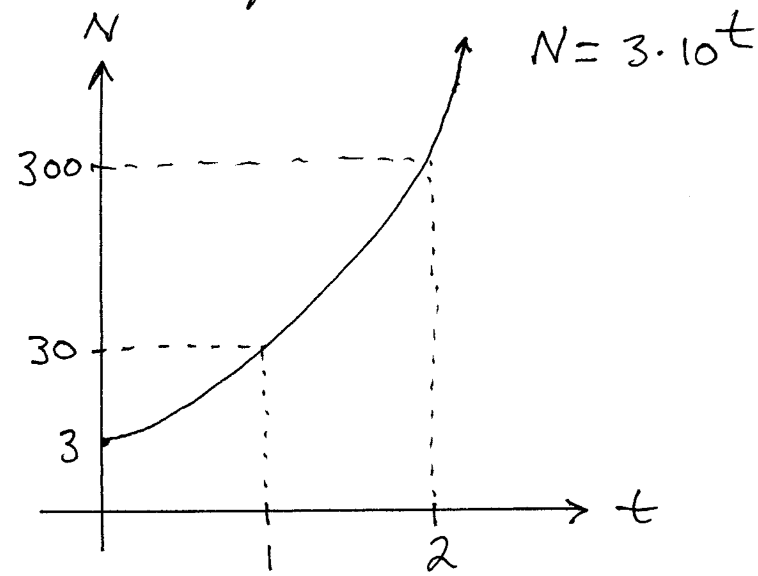
Math 17B
Kouba
Semi-Log Plots and Growth Rates

Ex: This example illustrates how some graphs plotted on a rectangular coordinate system become "linearized" when plotted on semi-log graph paper

1.) Plot the graph of $N = 3 \cdot 10^t$:



2.) Consider the following t and N values :

t	N
0	3
0.5	9.5
1	30
1.5	94.9
2	300
2.5	948.7

3.) Plot the values in 2.) on semi-log graph paper :

SEE HANDOUT

<u>Note</u>: The values shown on the vertical axis are values of N, but the values plotted on the graph are actually values of $\log N$. For example, notice the values $N=1$, $N=10$, $N=100$, and $N=1000$ are equally-spaced. This is because their positions on the vertical axis are actually represented by :

$$\log 1 = 0,$$
$$\log 10 = 1,$$
$$\log 100 = 2,$$

and $\log 1000 = 3$, numbers which are equally-spaced.

4.) Verify that the semi-log graph is a line :

Begin with $N = 3 \cdot 10^t$

$\rightarrow \quad \log N = \log(3 \cdot 10^t)$

$\rightarrow \quad \log N = \log 3 + \log 10^t$

$\rightarrow \quad \log N = \log 3 + t \cdot \log 10$

$\rightarrow \quad \log N = (\log 10) \cdot t + (\log 3)$

and $\quad Y = m \cdot t + b$

is the equation of line, where $Y = \log N$, slope $m = \log 10$, and Y-intercept $b = \log 3$.

Def: If $N = N(t)$, then its growth rate is $N' = \frac{dN}{dt}$.

Def: If $N' = \frac{dN}{dt} = f(N)$, a function of N only, then we say ~~the growth rate is~~ in autonomous form.

Ex: $\frac{dN}{dt} = 3N$, $\frac{dN}{dt} = N^2 - N$ are in autonomous form

5.) Find the growth rate for $N = 3 \cdot 10^t$ in autonomous form:

$$N = 3 \cdot 10^t \xrightarrow{D} \frac{dN}{dt} = 3 \cdot 10^t \cdot \ln 10$$

$$\to \frac{dN}{dt} = \ln 10 \cdot (3 \cdot 10^t)$$

$$\to \frac{dN}{dt} = \ln 10 \cdot N$$